Supplementary material to

Control of the size and compositional distributions in a milling process by using a reverse breakage matrix approach

Nemanja Bojanić¹, Aleksandar Fišteš¹, Tatjana Došenović¹, Aleksandar Takači¹, Mirjana Brdar¹, Kiyoshi Yoneda² and Dušan Rakić¹

¹Faculty of Technology, University of Novi Sad, Boulevard cara Lazara 1, 21000 Novi Sad, Serbia

Coefficient functions given in eq. (11) expressed by elements of matrices B and Y and contents of ash in input fractions are:

$$\begin{split} A_{12} &= b_{12}b_{23} - b_{13}b_{22} \,, \ A_1 = b_{13}Y_{22}P_2 - b_{12}Y_{23}P_3 \,, \\ A_2 &= b_{22}Y_{13}P_3 - b_{23}Y_{12}P_2 \,, \ A_0 = P_2P_3(Y_{12}Y_{23} - Y_{13}Y_{22}) \,, \\ C_{12} &= b_{13}b_{21} - b_{11}b_{23} \,, \ C_1 = b_{11}Y_{23}P_3 - b_{13}Y_{21}P_1 \,, \\ C_2 &= b_{23}Y_{11}P_1 - b_{21}Y_{13}P_3 \,, \ C_0 = P_1P_3(Y_{13}Y_{21} - Y_{11}Y_{23}) \,, \\ B_{12} &= b_{11}(b_{22} - b_{23}) + b_{12}(b_{23} - b_{21}) + b_{13}(b_{21} - b_{22}) \,, \\ B_1 &= P_1Y_{21}(b_{12} - b_{13}) + P_2Y_{22}(b_{13} - b_{11}) + P_3Y_{23}(b_{11} - b_{12}) \,, \\ B_2 &= P_1Y_{11}(b_{23} - b_{22}) + P_2Y_{12}(b_{21} - b_{23}) + P_3Y_{13}(b_{22} - b_{21}) \,, \\ B_3 &= P_2P_3(Y_{12}Y_{23} - Y_{13}Y_{22}) + P_1P_3(Y_{13}Y_{21} - Y_{11}Y_{23}) + P_3P_3(Y_{11}Y_{22} - Y_{12}Y_{21}) \,. \end{split}$$

Coefficient functions given in eq. (15) expressed by elements of matrices B and Y and contents of ash in input fractions are:

$$\begin{split} A_{11} &= \frac{b_{12} - b_{13}}{Y_{11}P_1(b_{12} - b_{13}) - Y_{12}P_2(b_{11} - b_{13}) + Y_{13}P_3(b_{11} - b_{12})} \,, \\ A_{1} &= \frac{Y_{13}P_3 - Y_{12}P_2}{Y_{11}P_1(b_{12} - b_{13}) - Y_{12}P_2(b_{11} - b_{13}) + Y_{13}P_3(b_{11} - b_{12})} \,, \\ A_{0} &= \frac{Y_{12}P_2b_{13} - Y_{13}P_3b_{12}}{Y_{11}P_1(b_{12} - b_{13}) - Y_{12}P_2(b_{11} - b_{13}) + Y_{13}P_3(b_{11} - b_{12})} \,, \\ B_{11} &= \frac{b_{13} - b_{11}}{Y_{11}P_1(b_{12} - b_{13}) - Y_{12}P_2(b_{11} - b_{13}) + Y_{13}P_3(b_{11} - b_{12})} \,, \\ B_{1} &= \frac{Y_{11}P_1 - Y_{13}P_3}{Y_{11}P_1(b_{12} - b_{13}) - Y_{12}P_2(b_{11} - b_{13}) + Y_{13}P_3(b_{11} - b_{12})} \,, \\ B_{0} &= \frac{Y_{13}P_3b_{11} - Y_{11}P_1b_{13}}{Y_{11}P_1(b_{12} - b_{13}) - Y_{12}P_2(b_{11} - b_{13}) + Y_{13}P_3(b_{11} - b_{12})} \,. \end{split}$$

Coefficient functions given in eq. (20) expressed by elements of matrices *B* and *Y* and contents of ash in input and output fractions are:

$$\begin{split} A_1 &= \frac{(b_{12} - b_{13})p_1 + Y_{13}P_3 - Y_{12}P_2}{Y_{11}P_1(b_{12} - b_{13}) + Y_{12}P_2(b_{13} - b_{11}) + Y_{13}P_3(b_{11} - b_{12})}, \\ A_0 &= \frac{Y_{12}P_2b_{13} - Y_{13}P_3b_{12}}{Y_{11}P_1(b_{12} - b_{13}) + Y_{12}P_2(b_{13} - b_{11}) + Y_{13}P_3(b_{11} - b_{12})}, \\ B_1 &= \frac{(b_{13} - b_{11})p_1 + Y_{11}P_1 - Y_{13}P_3}{(b_{12} - b_{13})p_1 + Y_{13}P_3 - Y_{12}P_2}, \\ B_1 &= \frac{Y_{13}P_3 - b_{13}p_1}{(b_{12} - b_{13})p_1 + Y_{13}P_3 - Y_{12}P_2}. \end{split}$$



²Faculty of Economics, Fukuoka University, 8-19-1 Nanakuma, Jonan-ku, Fukuoka 814-0180, Japan