Global sensitivity analyses of a neural networks model for a flotation circuit

Manuel Saldaña[1,2], Luis Ayala[1], David Torres[3] and Norman Toro[2]

[1]Faculty of Engineering and Architecture, Universidad Arturo Prat, Antofagasta 1244260, Chile
[2]Departamento de Ingeniería Metalúrgica y Minas, Facultad de Ingeniería y Ciencias Geológicas, Universidad Católica del Norte, Antofagasta 1270709, Chile
[3]Department of Mining, Geological and Cartographic, Universidad Politécnica de Cartagena, Paseo Alfonso Xlll N°52, 30203 Cartagena, Spain

Abstract

Modeling of flotation processes is complex due to the large number of variables involved and the lack of knowledge on the impact of operational parameters on the response(s), and given this problem, machine learning algorithms emerge as an alternative interesting when modeling dynamic processes. In this work, different artificial neural network (ANN) architectures for modeling the mineral concentrate in a rougher-cleaner-scavenger (RCS) circuit based on the main process variables are generated (variables as the recovery of the rougher, cleaner and scavenger cells, along with disaggregated variables). Analysis of the global sensitivity was performed to study the importance of the individual and joint performances of the stages of the flotation circuit, reflected by sensitivity indicators that allow to infer the impact that the stages and operational parameters produce on the dependent variables (mineral concentrate in rougher, cleaner and scavenger cells, in addition to the global concentration in the RCS circuit). It should be noted that the ANN is a useful tool for modeling dynamic systems such as flotation, while sensitivity analysis shows that the operation of the three threads turns out to be crucial for the subsequent evaluation of the circuit, while the Unbundled variables that most interact with the overall recovery are gas flow rate, bubble and particle diameters, bubble velocity, particle density, and surface tension.

Keywords: modeling; machine learning; mineral processing; flotation; sulfides.

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1. INTRODUCTION

There is a depletion of high-grade minerals in the earth's crust [1]. For example, in 1900 the average copper grade was 4%, while today it is 0.5 %. Because of this, mining has had to increase its production levels to compensate for the drop in copper grades [2].

Within copper extractive processes, flotation is the most widely used worldwide, producing 80 % of the production of this commodity [3–7]. Flotation processes are a recurring alternative in the treatment of copper sulfide minerals, since the selective separation of minerals is achieved, based on their hydrophobic and hydrophilic properties [8–10]. The flotation process consists of a structure in which series-ordered flotation banks are pre-established [11,12], which cover the treatment of a large volume of material, which in turn produces associated costs. By analyzing the literature, it is possible to find studies that provide improvements in the behavior associated with flotation circuits [11,13], optimization techniques [14], or tools such as sensitivity analysis [15,16]. Sensitivity analysis, is defined as a study of how the uncertainty in the product of a model (numerical or otherwise) can be assigned to different sources of uncertainty at the input of the model, offering a wide spectrum of applications and identifying the most significant variables [14,16,17]. Likewise, sensitivity analyses can be separated into two major classifications, the global sensitivity

Corresponding author: Manuel Saldaña, Universidad Arturo Prat, Antofagasta 1244260, Chile; Tel.: +56953834174
E-mail: masaldana@unap.cl
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analysis, and the local sensitivity analysis [16,17]. The first is defined as the evaluation of the response when the independent variables of the model are evaluated simultaneously [12,16,17], while the second case is defined as the local measure of the effect of an input value for a given product [11,14,16,17].

Considering the above, and the complexity of the flotation process due to the number of variables involved, it is proposed to model dynamics of the process by machine learning techniques and, after generating the model, sensitizing the independent variables, in order to determine how they affect the dependent variable [18,19].

In the section materials and methods, the flotation process is defined, indicating the variables (explanatory and explained) considered for the study and the ranges of operation observed under experimental conditions (pilot plant). Modeling of the mineral concentrate was performed by means of a digital model using machine learning techniques (Artificial Neural Networks, ANNs), while an overview of the sensitivity analysis techniques used to measure the impact on the response of the independent variables is given. Then, in the Results and Discussions section, the adjustment and sensitivity of the models based on neural networks are presented, along with the effect of the stages of the rougher-cleaner-scavenger (RCS) circuit, such as the ungraded variables in the dependent variable. Finally, the Conclusions section summarizes the main findings of the work carried out.

2. MATERIALS AND METHODS

2. 1. Flotation process

Flotation is defined as a process of mineral concentrate in which attempts are made to separate the useful ore particles from gangue, by means of a physical-chemical treatment [20]. This process is also called froth flotation, and the essential flotation mechanism involves the attachment of mineral particles to air bubbles, such that these particles are brought to the surface of the mineral pulp, where they can be removed [20]. The circuit considered in the present study consists of rougher, scavenger and cleaner flotation stages [11,21], like the one shown in Figure 1.

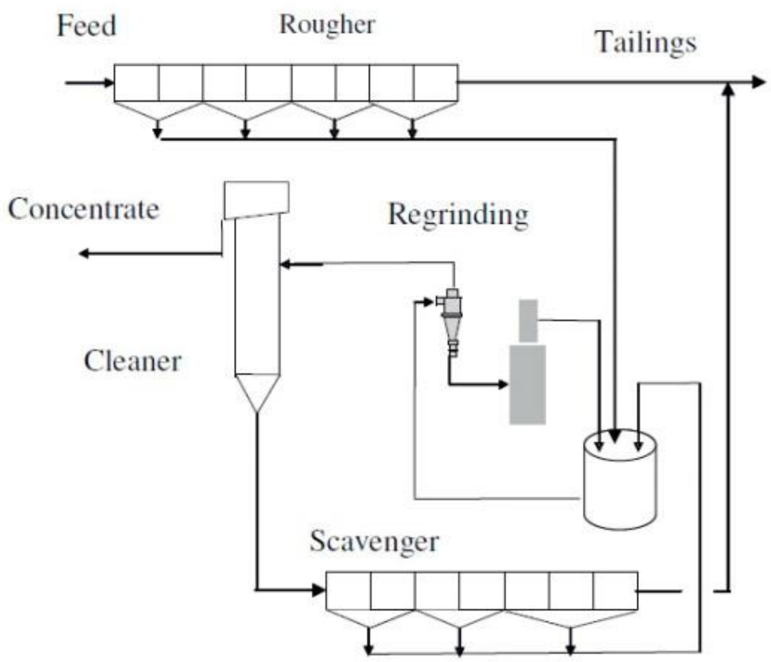

Figure 1. Structure of a high-level flotation circuit.

The rougher-cleaner-scavenger (RCS) circuit is simulated to evaluate the variation of the flotation performance by developing models based on machine learning techniques, while the variables considered in the modeling are presented in Table 1.

Table 1. Summary of independent variables in each cell of an RCS circuit.

Independent variable	Lower limit	Upper limit
Mean resident time in the cell, τ_M / min	20	30
Gas flow rate, G_{fr} / cm^3 min^{-1}	3250	3750
Bubble size, d_b / cm	0.05	0.10
Particle size, d_p / µm	1	100
Bubble speed, V_b / cm s^{-1}	10	30
Turbulence dissipation rate, ε / m^2 s^{-3}	18	30
Kinematic fluid viscosity, v / cm^2 s	0.008	0.01
Bubble kinematic viscosity, v_b / cm^2 s	0.008	0.01
Particle density, ρ_p / kg m^{-3}	1.3	4.1
Fluid density, ρ_f / kg m^{-3}	0.9	1
Surface tension, σ / N m^{-1}	68	75

The limits of the operational parameters shown in Table 1 correspond to the domain of the flotation process (in each of the rougher, cleaner and scavenger cells) in a pilot plant of a mine in Chile's Antofagasta region.

The speed of the bubbles is assumed to be inversely proportional to the size of the bubbles, that is, small bubbles rise slower than large bubbles, while in the presence of different solutes small bubbles rise slower than just in water [22]. The flotation rate, and therefore, the mineral concentrate is directly proportional to the gas flow rate for particle sizes > 10 µm (for finer particles it is not possible to affirm the direct proportionality or determine the deviation from proportionality) [23]. Flotation requires a certain degree of turbulence for a variety of reasons, including maintaining the solids suspended in the pulp phase, introducing air into the pulp and dispersing it into bubbles, mixing the aerated pulp for distribution and conditioning of reagents, as well as providing opportunities for collision of particles with bubbles [24]. Turbulence dissipation rate is directly related to the speed of the impeller (among others), and the higher the speed of the impeller, the greater the separation of solid particles from air bubbles, and therefore the lower mineral recovery [23]. Thus, the higher the turbulence dissipation rate, the lower probability of maintaining solids in suspension, while the lower dissipation rate, the greater probability of recovery of solid particles in the froth.

Chemical modifiers are chemical reagents that alter flow properties of the fluid and in particular the suspension of solids in liquid media [25]. Thus, handling the response depends mainly on the type and amount of chemical additives used to modify the flow properties. Density of the particle must be greater than that of the fluid, which is confirmed in literature [26]. A study carried out by Mehrotra and Kapur [27] indicates that the lower the pulp density, the higher the flotation ratio, and therefore, the higher mineral concentrate. Other studies show that the pulp density affects the key parameters of the pulp and froth phases, since the flowrates of the pulp phase decrease at high density, which is attributed to a reduction of turbulence within the cell [28]. Then, the particle size is one of the most important parameters in flotation, and three size classes are recognized determining the recovery dynamics, an intermediate size range where the recovery is high; a fine size range where the initial recovery is lower and increases over time; and a coarse size range where the recovery is again lower but less affected by time. The size divisions are not precise and will vary with the type of mineral [29].

Finally, the surface tension depends directly on the frothing agents used in the process [30], so that the higher the surface tension, the lower the froth amount. However, surface tension dependence is multifactorial, being influenced by other operational parameters, such as the types of frothing and collector agents and the amounts added.

2. 2. Machine learning

Machine Learning pertains to the creation of algorithms or computer programs that automatically improve and/or adapt their performance through experience, sharing features in common with other domains, such as statistics and probability theory, data mining and cognitive science [31]. The objective of machine learning is to devise learning algorithms that learn automatically, generating abstractions from a process by which the computer creates its own program based on a set of samples [32]. In order to explore and understand a process, it is necessary to try to quantify the events associated with its variables, which often follow a sequential evolution that could converge in one or more patterns, which generally determine the behavior and/or dynamics of the process.

Machine learning types can be classified into three categories: supervised learning, unsupervised learning, and reinforcement learning depending on the nature of the resulting data. In supervised learning, algorithms work with tagged data, trying to identify a function that, given the input variables or independent variables, assigns them the output tag or the appropriate response. The algorithm is trained with a data history and thus learns to assign the appropriate output tag to a new value, it has the ability to generalize based on a known universe of samples [33]. Unsupervised learning, on the other hand, takes place when the tagged data for the training is not available, only the knowledge of the input data is available, while the output data corresponding to a given input or set of inputs are not known Therefore, it is only possible to describe the structure of the data, trying to find a type of organization that simplifies the analysis, in other words, unsupervised learning is mandatory [34]. Finally, reinforcement learning is based on improving the model's response using a feedback process, the algorithm learns by observing the system in which it is inserted and its input information is the feedback it receives from the outside world in response to its actions, that is, the system learns based on trial and error [35–37].

Then, considering the nature of the dynamics of the flotation process and the existence of input data with their respective labels, the sensitivity analysis using artificial neural networks (ANN) is proposed. ANNs have the ability to handle large data sets, approximate nonlinear relationships, and generalize complex systems from relatively imprecise information [38]. In a system with ANN, the nodes are connected by means of synapses, and this connection structure determines the behavior of the network, with the multilayer perceptron being the mostly used structure [39], as shown in Figure 2.

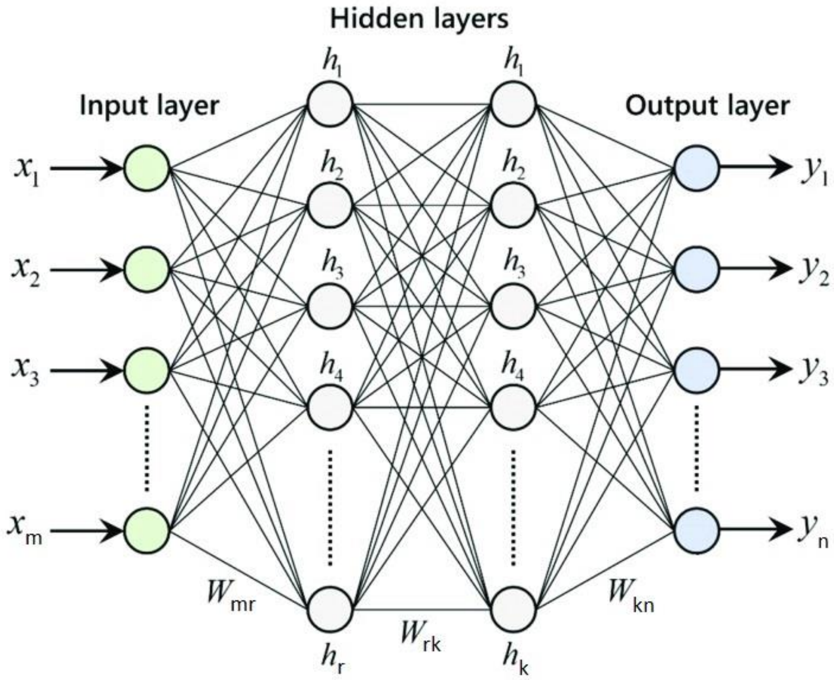

Figure 2. Multilayer perceptron architecture

In the ANN models, the input layer depends on the information available to be classified (x_1 to x_m, all variables in Table 1), the output layer is the mineral concentrate in froth in the Rougher, Cleaner and Scavenger cell by each ANN, and *W* are the synaptic weights corresponding to each entry. The neurons in one layer are connected to those in the next layer by synapses and the connection values (or synaptic weights) are determined through the training process, considering the activation functions of the neurons and the initial values (random) of the connections [38]. The modeling and simulation of neural network models is carried out by using the Keras library [40] in Python [41], the learning method used is backpropagation, and the activation functions of the network neurons are: "relu" (Rectified Lineal Unit) for the neurons of the hidden layer and "sigmoid" for the output layer.

2. 3. Sensitivity analysis method

Two methods based on the calculation of the variances were used: the Fourier extended method (FAST) and the Sobol method [18,42] to obtain the first order sensitivity indices and the global sensitivity indices. The first order sensitivity index (S_i), represents the contribution of each input factor (X_i) to the total variance of the output ($V(Y)$), and is denoted by the following equation:

$$S_i = \frac{V\left[E\left(Y|X_i\right)\right]}{V\left(Y\right)} \tag{1}$$

Where $V[E(Y|X_i)]$ is the amount of expected variance due to the main effect that would be removed from the total variance $V(Y)$ if the true value of factor X_i could be determined. Then, the total unconditional variance of the output variable Y is presented as:

$$V(Y) = E[V(Y|X_i)] + V[E(Y|X_i)] \tag{2}$$

Where $V[E(Y|X_i)]$ is the expected amount of variance of the response that would remain unexplained if the factor X_i were released over its range of uncertainty. The total effect sensitivity index (S_{Ti}, Eq. 4) that accounts for the total contribution of the output Y due to the factor X_i that is, its first-order effect plus all the higher-order effects resulting from the interactions, is obtained by rewriting Eq. (2) conditioning with respect to all factors except $X_{\sim i}$:

$$V(Y) = E[V(Y|X_{\sim i})] + V[E(Y|X_{\sim i})] \tag{3}$$

$$S_{Ti} = 1 - \frac{V\left[E\left(Y|X_{\sim i}\right)\right]}{V\left(Y\right)} = \frac{E\left[V\left(Y|X_{\sim i}\right)\right]}{V\left(Y\right)} \tag{4}$$

By definition $S_{Ti} > S_i$ or $S_{Ti} = S_i$ when X_i does not participate in interactions with another factor. The differences $S_{Ti} - S_i$ and $1 - \Sigma_i S_i$ are a measure of the participation of X_i in interactions with another factor. If $S_{Ti} = 0$, it means that the model is not sensitive to this factor [17]. The sum of all S_i equals 1 when the model is additive and less than 1 if it is non-additive. The sum of the S_{Ti} is greater than 1 for non-additive models and equal to 1 only in the case of additive models [18].

3. RESULTS AND DISCUSSION

3. 1. Adjustment and sensitivity of the neural network model

Before the sensitivity analysis using ANNs, a model must be generated adjusting the operational data of a flotation circuit (RCS), for which different network architectures were generated and their adjustment was studied by using the goodness indicators of fit: Mean Absolute Deviation (MAD), Mean Squared Error (MSE), Accuracy (ACC) and R^2 [43,44]. The training and testing of 4 networks were considered, with network architectures of one and two hidden layers and three and five neurons per hidden layer. Each network architecture was trained and tested by dividing the dataset into a 70 : 30 ratio (data provided by the national mining company ENAMI, Antofagasta, Chile), while the dataset size is approximately 1800 samples.

The set of input variables of the artificial neural network models present the mean resident time in the cell, gas flow rate, bubble size, particle size, bubble speed, turbulence dissipation rate, kinematic fluid viscosity, bubble kinematic viscosity, particle density, fluid density and surface tension. On the other hand, the response variables are defined as the mineral concentrate in the froth zone for the rougher, cleaner and scavenger cells.

The statistics of the relative error percentage between the predicted responses and the measured values are summarized in Table 2.

From the analysis of the goodness-of-fit statistics presented in Table 2, the configuration that presents a better fit in the training stage and a lower dispersion of errors is the configuration of 2 layers, 5 neurons per layer for the rougher cell and a configuration of 2 layers and 3 neurons for the cleaner cell, while the 1-layer and 5-neuron-per-layer architecture is the one suited best for the scavenger cell.

Table 2. Goodness-of-fit statistics of the ANN architectures for the cells in the RCS circuit

	Architecture/Statistic	Training				Testing			
	(layer, neurons)	MAD	MSE	ACC, %	R^2, %	MAD	MSE	ACC, %	R^2, %
Rougher	1 layer, 3 neurons	0.0345	0.0009	85.72	72.83	0.0373	0.00101	79.17	65.80
	1 layer, 5 neurons	0.0263	0.0009	91.51	70.98	0.0288	0.0009	82.05	65.98
	2 layers, 3 neurons	0.0201	0.0008	89.25	77.29	0.0210	0.0008	79.34	68.25
	2 layers, 5 neurons	0.0194	0.0006	92.47	85.69	0.0209	0.0006	81.92	75.58
Cleaner	1 layer, 3 neurons	0.0380	0.0010	77.50	64.19	0.0390	0.0011	68.16	62.23
	1 layer, 5 neurons	0.0283	0.0009	82.75	64.06	0.0298	0.0010	75.89	61.56
	2 layers, 3 neurons	0.0209	0.0007	82.50	78.40	0.0238	0.0007	74.37	71.14
	2 layers, 5 neurons	0.0219	0.0009	82.89	72.77	0.0234	0.0010	75.20	66.52
Scavenger	1 layer, 3 neurons	0.0410	0.0010	70.15	57.93	0.0427	0.0012	63.45	51.87
	1 layer, 5 neurons	0.0229	0.0007	76.77	69.64	0.0258	0.0008	71.28	61.49
	2 layers, 3 neurons	0.0301	0.0010	73.70	55.86	0.0299	0.0011	68.98	50.28
	2 layers, 5 neurons	0.0248	0.0010	75.83	61.42	0.0257	0.0010	71.56	57.05

The performance of the training and test data sets is presented in Figure 3.a, while the variation of the coefficient of determination throughout the training and testing periods is shown in Figure 3.b. Additionally, the ANN model is validated by contrasting the optimal mineral concentrations generated by the fitted model and the operational data (see Fig. 4). The scenarios above the red line represent those configurations that have a higher theoretical concentration.

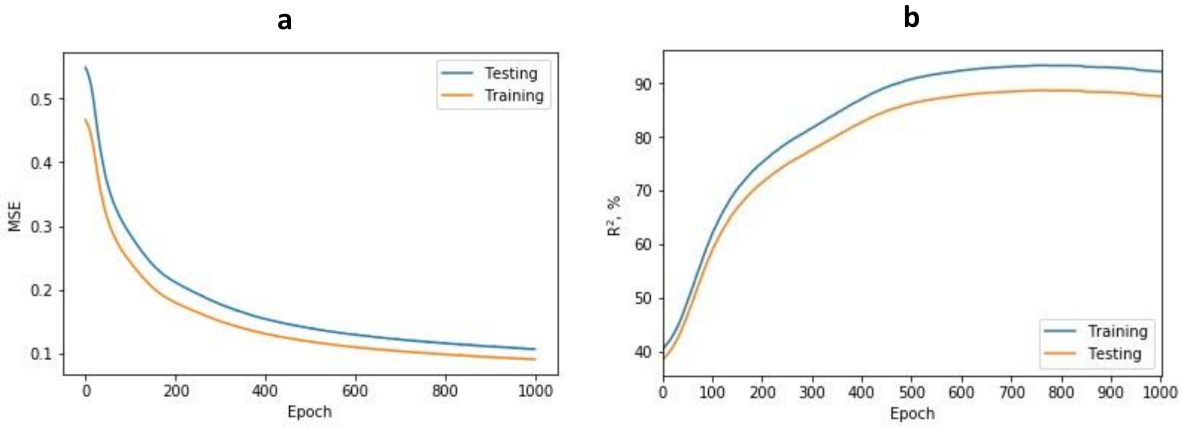

Figure 3. MSE (a) and determination coefficient (b) for training and testing phases of the adjusted model for the rougher cell (Architecture: 2 layers, 5 neurons)

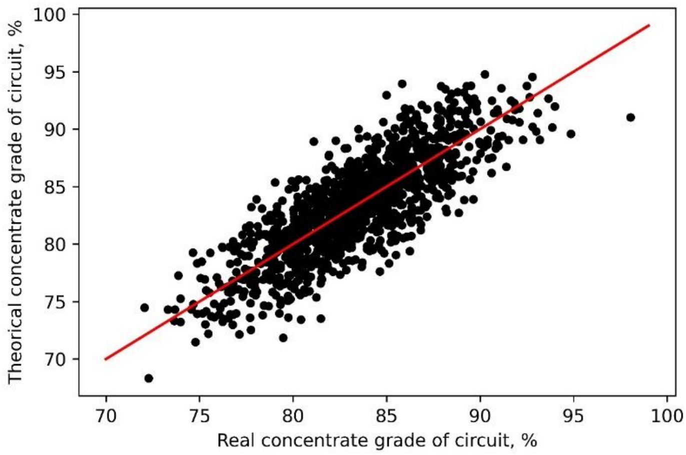

Figure 4. Real versus predicted mineral concentrate of the RCS circuit (output of the cleaner cell, Fig. 1)

3. 2. Effect associated with the degree of uncertainty of the ANN model

As a result of the sensitivity analysis of the ANN model, the individual effect produced by each added stage of an RCS circuit can be seen, that is, the sensitivity analysis of the mineral concentrate in the rougher (R_R), cleaner (R_C) and scavenger (R_S) cells, as well as the effects of the parameters that affect the mineral concentrate within these cells, disaggregating the aggregate process, and analyzing the impact on the global mineral concentrate of the following variables: mean resident time in each of the cells (τ_M), gas flow rate (G_{fr}), bubble size (d_b), particle size (d_p), bubble speed (V_b), turbulence dissipation rate (ε), kinematic viscosity of the fluid (ν) and the bubble (ν_b), particle density (ρ_p) and fluid density (ρ_f), induction time (t_{ind}) and surface tension (σ). Then, by quantifying the effect of the independent variables on the dependent variables (by the first-order and total effect index), it is expected to have a greater understanding of the stages more critical when searching for an optimum or to improve the operational results [45–51], such as the overall mineral concentrate at the output of the RCS circuit.

Figure 5 shows that the operation of the three threads turned out to be crucial for the subsequent evaluation of the circuit presented in Figure 1. The total indices of the most influential parameters, both of the aggregate models that represent the threads of the RCS circuit, as well as the disaggregated parameters, present the greatest differences S_{Ti} - S_i, which indicates that these parameters affect to a greater degree the state variable along with other parameters, participating in turn in the interactions. The disaggregated variables that most interact are the gas flow rate, diameters of the bubble and the particle, the speed of the bubbles, the particle density and the surface tension, as can also be seen in Figure 5, while the Sobol method (see Figure 6) confirms the results obtained with the FAST method.

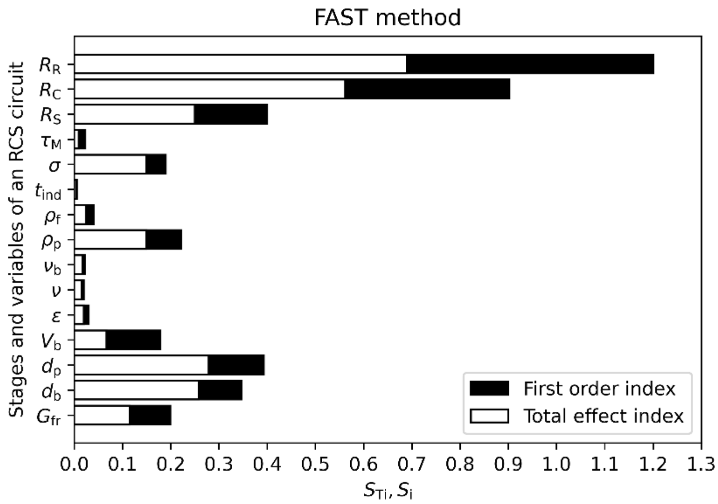

Figure 5. Average sensitivity indices calculated by the FAST method in the RCS circuit.

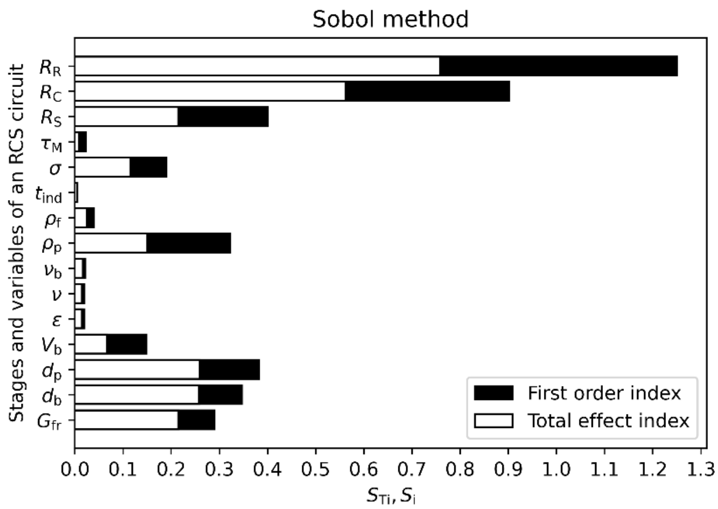

Figure 6. Average sensitivity indices calculated by the Sobol method in the RCS circuit.

4. CONCLUSIONS

In the development of the study, the importance of individual and group performance of the stages that comprise a flotation circuit is appreciated, reflected by sensitivity indicators that allow to infer the degree of impact that the stages produce on the dependent variables, after modeling the dynamics of the flotation circuit process using a system of artificial neural networks. By considering these indicators and the results obtained, it is possible to identify that the variables: bubble size (d_b), particle size (d_p), particle density (ρ_p), gas flow rate (G_{fr}) and surface tension (σ) have the greatest impact on the performance of the circuit, and should be taken into account when generating methodologies for increase the performance of the system.

The ANN model presents a good fit to the modeled system, validated by the goodness-of-fit indicators and by the comparison of real versus predicted mineral concentrates. This work provided possibilities to appreciate how sensitivity analysis can be applied to models based on machine learning, by modeling a flotation circuit, instead of evaluating each parameter operationally. The behavior of the stages (cells) and individual parameters was evaluated to understand which stages and/or operational parameters turn out to be the key to the fulfillment of the operational objectives. On the other hand, in future works, the dynamics of the system could be modeled using other machine learning techniques [52–54], and the developed models could be incorporated into a simulation framework [46] that quantifies the benefits associated with improving the efficiency of the process, which can lead to pilot tests and the implementation of this type of predictive models in production processes.

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SAŽETAK

Analiza osetljivosti modela neuronskih mreža za flotacioni sistem

Manuel Saldaña[1,2], Luis Ayala[1], David Torres[3] i Norman Toro[2]

[1]Faculty of Engineering and Architecture, Universidad Arturo Prat, Antofagasta 1244260, Chile
[2]Departamento de Ingeniería Metalúrgica y Minas, Facultad de Ingeniería y Ciencias Geológicas, Universidad Católica del Norte, Antofagasta 1270709, Chile
[3]Department of Mining, Geological and Cartographic, Universidad Politécnica de Cartagena, Paseo Alfonso Xlll N°52, 30203 Cartagena, Spain

(Stručni rad)

Modelovanje procesa flotacije je složeno zbog velikog broja promenljivih koje su uključene u ovaj proces i nedovoljnog znanja o uticaju operativnih parametara na dobijeni rezultat. Imajući u vidu ove probleme, algoritmi mašinskog učenja su zanimljiva alternativa prilikom modelovanja dinamičkih procesa. U ovom radu su formirane veštačke neuronske mreže različitih arhitektura (engl. artificial neural network, ANN) za modelovanje mineralnog koncentrata u flotacionom sistemu sa osnovnom i dopunskom flotacijom (engl. rougher-cleaner-scavenger, RCS) na osnovu glavnih promenljivih procesa (promenljive poput izdvajanja minerala u pojedinačnim flotacijskim jedinicama, kao i individualne promenljive). Analiza osetljivosti izvedena je kako bi se proučila važnost pojedinačnih i zajedničkih performansi jedinica flotacionog sistema, izražena indikatorima osetljivosti, koji omogućavaju određivanje uticaja samih jedinica i radnih parametara na zavisne promenljive (količina mineralnog koncentrata u svakoj od jedinica sistema, uz ukupnu izlaznu koncentraciju u flotacionom sistemu). ANN su se pokazale kao koristan alat za modelovanje dinamičkih sistema poput flotiranja. Analizom osetljivosti pokazano je da je rad tri jedinice presudan za procenu efikasnosti rada flotacionog sistema, dok nezavisne promenljive koje najviše utiču na ukupnu ekstrakciju minerala jesu protok gasa, prečnici mehurova gasa i čestica, brzina mehurova, gustina čestica i površinski napon.

Ključne reči: modelovanje; mašinsko učenje; izdvajanje minerala; flotacija; sulfidi