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3	Decay of the melt stream during dispersion in granulation devices
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ABSTRACT

25 The aim of the article is a theoretical description and experimental study of the melt jet expiration 26 process from a perforated shell. Mathematical modeling of hydrodynamic flows was carried out based on the points of classical fluid and gas mechanics and technical hydromechanics. 27 28 Mathematical model equations were solved by using computer mathematics of Maple and 29 wxMaxima. Reliability of the obtained experimental results is based on the application of time-30 tested in practice methods. Hydrodynamic properties of the liquid jet outflow were obtained. The 31 presented mathematical model allows calculation of radial component of the jet outflow velocity, 32 as well as determination of the influences of physical and chemical properties of the liquid and the 33 outflow hole diameter on the jet length and flow velocity along the axis to its disintegration into 34 separated drops. The developed mathematical model extended with the theoretical description 35 of the melt dispersion process from rotating perforated shells allowed us to improve design of the 36 granulator to stabilize hydrodynamic parameters of the melt movement. Consideration of 37 hydrodynamic parameters of the fluid jet flowing out of the holes of the perforated membrane 38 provides improvement of the construction of nitrogen fertilizers melt dispersant, affect to the 39 parameters of the process of jet decay into drops, its size and monodispersity. Basket tests of the 40 granulator was confirmed the theoretical research and provided a basis for modernization of 41 the equipment construction. Keywords: jet decay, vibration granulator, hydrodynamic of flows movement, melt 42 dispersion process, rotating perforated shell 43

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1. INTRODUCTION

Liquid dispersion processes forming micro- or macro drops are used in power generation,

medicine, chemical industry, agriculture and other spheres of human activity. Efficiency of these 48 technological processes and equipment is largely determined by the quality of liquid dispersion, 49 50 which usually involves obtaining monodisperse drops [1]. 51 This fully applies to the production of the commodity form of nitrogen fertilizers, which is 52 carried out in two main ways [2]: 53 - granulation starting from the liquid phase by dispersion on the surface of suspended particles in a fluidized bed that can be variously configured (technologies of Casale S.A., Switzerland; Kahl 54 55 Group, Germany; Stamicarbon, Netherlands; Toyo Engineering Corporation, Japan; 56 Thyssenkrupp Fertilizer Technology GmbH, Germany, etc.) [3-8], including vortex granulation [9, 57 10]; - granulation starting from the liquid phase by dispersion into drops followed by crystallization 58 of the solute by dewatering and cooling (prilling) (devices of Norsk Hydro, Norway; Didier 59 60 Engineering GmbH, Germany;, Imperial Chemical Industries, UK; Kaltenbach-Thuring S.A., 61 France et al.) [11]. 62 In these methods, among others, devices with different forms of a perforated shell, generally being axially symmetrical can be used for dispersion of the nitrogen fertilizer melt. 63 64 Devices for melt dispersion can be classified by the form of the working part (*i.e.* perforated 65 shell) and by the presence of internal devices in the perforated shell. Additionally, these devices 66 differ in the nature of force acting on the melt and can be static, swirl (tangential introduction of 67 the melt into a perforated shell or to the turbine for the melt spin), and dynamic (rotating) [12].

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In recent years, preference is mainly given to conical or cylindrical rotating devices and devices
with the cup-shaped shell. This is due to simplicity of operation and high uniformity of resulting
liquid drops and commodity granules in comparison to analogue devices.

For example, ammonium nitrate granulators (dispersers), which are currently in operation, provide manufacturing of products with following granulometric composition in terms of mass fraction: 0.5 - 1.5 % of granules < 1.0 mm in size, 90 – 98 % of granules in the size range 2.0 - 4.0 mm, where granules in the size range 2.0 - 2.5 mm comprise 42 - 71% and granules in the size range 2.0 - 3.0 mm comprise 85 - 95% [13, 14]. Dispersion of melts producing more than 2 % of dust-forming particles of less than 1.0 mm and over 3.5 mm in size, which can be also destructed making dust, leads to dust formation of nitrogenous fertilizers in air in the tower.

In existing equipment calculation of hydrodynamic characteristics of the liquid jet that is dispersed is often not performed, resulting in lower uniformity of the obtained drops [15]. Hydrodynamic parameters of the liquid jet issuing from a single hole or holes of the perforated shell, and design features of devices for fluid dispersion influence the process of jet decay into drops.

The problem of creating the adequate model of jet decay into drops at the opening in a thin wall is highly relevant for dispersion improvement in granulation devices in the mineral fertilizers production. By controlling the jet decay process, we can optimize the performance of dispersant and create favorable conditions to produce a product with a high degree of monodispersity. The aim of the article is a theoretical description and experimental study of the melt jet outflow

88 process from a perforated shell (basket).

2. DESCRIPTION OF THE OBJECT AND METHODS OF RESEARCH

91 Mathematical modeling of hydrodynamic flows was carried out based on the postulates of classical fluid and gas mechanics and technical hydromechanics [16, 17]. Solving equations of 92 93 mathematical models was carried out by using the Maple software (Maplesoft, Canada) [18] 94 and the open-source computer algebra system wxMaxima [19, 20]. These systems have proven 95 to provide reliable and efficient symbolic and numerical algorithms for a wide range of mathematical problems, including a well-known library of numerical algorithms NAG (Numerical 96 97 Algorithms Group, UK). Adequacy of calculated dependences for the process that was 98 investigated, is validated by the comparison of calculated data with known and experimentally 99 obtained results.

100 2.1 Experimental vibrating granulator

A vibrating granulator (manufacturer – Sumy State University, Processes and Equipment of Chemical and Petroleum-Refineries Department, Ukraine; granulator diameter is 560 mm, granulator height is 590 mm) is the main unit of the experimental unit (Fig. 1), which consists of a variable perforated membrane 1 with holes to discharge the fluid, housing 2 with a distributive drive 3 and a pipe 4 for introducing air from the fan 5. The housing 2 is also supplied with a fixed liquid distributor 6 with a pipe 7 and a filter element 8.

107 At the top of the fluid atomizer, a mechanical, electrical or electromagnetic vibrator 9 is 108 installed, which is connected by a rod 10 with the resonator 11 in the form of an elastic disc or 109 plate.

110 When the granulator is working, liquid that goes through the pipe 7 and the distributor 6 to the 111 bottom part of the granulator flows out of the holes of the perforated membrane. 112 Simultaneously, air at a given pressure is supplied by the fan 5 through the pipe 4 into the cavity

113 of the granulator.



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Fig. 1. Experimental installation of the vibrating granulator. a) A schematic presentation of the granulator (design features of elements which are not described – see [21]): 1- perforated membrane (diameters of holes are 1.0 mm, 1.1 mm, 1.2 mm, 1,3mm, 1,4 mm, number of holes is 1800-2300, bottom shape is toroidal, length is 650 mm, diameter is 560 mm), 2 – housing (height is 590 mm, diameter is 560 mm), 3 – distributive drive, 4 – pipe (diameter is 123 45 mm), 5 – fan, 6 – liquid distributor, 7 – pipe (diameter is 100 mm), 8 - filter element (metal 124 grid), 9 – vibrator (MFR OTY 77 actuator, range of output frequency 120-1200 is Hz), 10 – rod, 125 11 - resonator, 12 - buffer bunker, 13 – circulation pump (model Calpeda NC3 25-50/180), 14 – 126 valve, 15 – rotameter (model Raifil RF FM 10), 16 - oscillations sensor, 17 – oscillograph 127 (model C1-65A), 18 – digital frequency meter (model VC3165), 19 – electronic controller, 20, 128 21 – elements of stroboscope, 22, 23 – manometers (model MT-2y), 24 – controller (model 129 Euroaqua SKD-1); b) 3D model of vibrating granulator; c) 3D model and photo of experimental 130 **installation**

131 The installation is equipped with a buffer bunker 12 for liquid with a circulation pump 13. Valve 14 132 and rotameter 15 are used to regulate and measure the liquid flowrate. The oscillations sensor 16 133 is connected with an oscillograph 17 to observe fluctuations while the digital frequency meter 18 is 134 used to measure the oscillation frequency. When the electrodynamic or electromagnetic vibrator is 135 used, regulation of oscillations is carried out by the electronic controller 19. A stroboscope (20 and 136 21) is installed for visual observation of the process of liquid dispersion into drops. Measurements 137 of the liquid pressure in holes and air at the free surface of the liquid are carried out by 138 manometers 22 and 23, respectively. The controller 24 is aimed to regulate the air pressure in the 139 granulator.

Next, the electronic oscillator 19 is switched on and the electrodynamic vibrator 9 started resulting in vibration of the resonator at a certain frequency, which is fixed for the testimony of digital frequency meter 18, which is connected to the vibration sensor 16 (measurer of frequency) of the granulator basket. Air is supplied into the granulator volume by turning the fan 5 on, then when the interstitial position is changed, the given pressure is installed based on manometer data. The process of jets dispersion into drops is simultaneously monitored by using a stroboscope 20. When formation of monodisperse drops (without satellite droplets) is

- 147 observed, measurements are recorded at the manometer 22, which corresponds to the total
- 148 liquid pressure in the leakage holes, and at the digital frequency meter 18 showing the vibration
- 149 frequency (upper limit). These liquid and air parameters are changing by changing the vibration
- 150 frequency with electronic generator 19 by the lower limit of granulator stable operation range.
- 151 Ranges of stable operation are determined for different liquid leakage velocities from holes in
- 152 the basket, which can be achieved by changing the granulator performance under constant air
- 153 pressure or by changing the air pressure by using the pressure regulator at the constant
- 154 granulator performance. When operating the vibration granulator at different liquid leakage
- velocities, the fluid flow rate is measured by a graduated cylinder.
- 156 Physical modeling is based on methods of the similarity theory. The geometric similarity is
- 157 maintained by equality of appropriate constants and invariants [22].
- 158 The special frequency generator is designed to generate vibration on the radiator of rotating
- 159 vibrational melting granulator by feeding electric signals of a special shape on the MFR OTY 77
- 160 actuator (system operation description in accordance with [23]).
- 161 Main parameters and characteristics:
- 162 frequency range of reproducible oscillations, Hz 200 1000
- 163 rated load, W 50 100
- limits of the permissible relative basic error, setting of the oscillation frequency, % 0.5
- voltage of the mains supply of the generator, V 220±10
- 166 frequency of the mains, Hz 60
- 167 active resistance of the moving coil, Ohm 8
- 168 established trouble-free operating time, h 1000 (with a confidence probability 0,95)
- 169 Operating conditions:
- 170 ambient temperature -10 + 70 ° C
- 171 relative humidity 50 80% at a temperature of +25 °C

- 173 The special frequency generator consists of:
- Generator of electrical signals of a special form
- 175 Actuator MFR OTY 77
- 176 The generator of electrical signals of a special form consists of:
- Block of digital synthesis of frequency
- 178 The amplifier
- 179 Power supply unit
- 180 Specifications of the electrical signal generator:
- 181 frequency range 200 1000 Hz
- 182 the minimum frequency setting step is 0.01Hz
- 183 output power 60W
- 184 load resistance 3.7 Ohm
- 185 harmonic coefficient at the frequency of 500 Hz 0.1%
- 186 load inductance 7.3 mH
- 187 The digital synthesis unit performs direct digital frequency synthesis (DDS Direct Digital
- 188 Sinthesizer) and is implemented on a microprocessor. Due to this, the generator has a high
- 189 frequency stability and a small step of its tuning. The unit allows you to store 10 preset
- 190 frequencies in the non-volatile memory. Each of which can be easily changed during operation.
- 191 Indication of the generated frequency is carried out on the liquid crystal display.
- 192 The amplifier provides the necessary amplification of signals from the generator to feed them
- 193 into a low-resistance load. The amplifier is implemented on an integrated microcircuit. The
- 194 output stage is made on field-effect transistors. This ensures a high efficiency amplifier. The
- 195 amplifier has a built-in protection system against overload, overheating and "soft start" system.
- 196 To power the whole device a power supply made by the classical transformer scheme is used.
- 197 Two bipolar rectifiers supply a +/- 12V the circuit for digital synthesis of frequency, and +/-
- 198 24V amplifier.

199 <mark>/</mark>	Actuator I	MFR	OTY 77
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- 200 Electromagnetic vibrator consists of control block and vibro-converter. Control block is used for
- 201 delivery of a signal of set frequency and amplitude to vibro-converter. A signal received from
- 202 the control block, is transferred to the vibro-converter where under influence of magnetic field
- 203 a reorientation of crystal lattice of the core alloy material occurs. As a result the core changes its
- 204 length. The radiator of granulator's vibrations is joined to the bottom part of the core. Cooling
- 205 of vibro-converter is provided by cooling air expulsion. The vibro-system has an input and
- 206 output signal 4-20 мА what will allow to automatically control the frequency of vibration
- 207 depending on the change of the fusion level in granulator. Control block of the vibro-system is
- 208 fixed on CPU.
- 209

- 2.2 Processing of the experimental results
- 211 Velocity, V, of the liquid leaking from the granulator holes is calculated as:
- 212 $V = \phi \cdot \sqrt{2 \cdot g \cdot H}$. (1)
- where ϕ is the discharge coefficient and set to 0.96-0.98 [24].
- According to experimental data of upper f_1 and lower f_2 limits of the frequency, which provides
- a monodisperse liquid jet decay, the maximum and minimum lengths of the wave are
- 216 calculated:
- 217 $\lambda_{max}=V/f_1; \lambda_{min}=V/f_2,$ (2)
- 218 The melt flowrate, G_s, through the granulator hole is calculated from the measured leaked melt
- 219 volume, G_{τ} and time, τ :
- 220 $G_s = G_{\tau} / \tau$. (3)

221 Diameter of drops, formed by the decay of liquid jets at the average vibration frequency of the

granulator, f_{avi}, is determined from the material balance according to the equation:

223
$$d_{dr} = 3 \sqrt{\frac{6 \cdot G_s}{\pi \cdot f_{avi}}}.$$
 (4)

The absolute difference of drop diameters and a relative deviation of drop diameters from the average drop diameter at the granulator maximum and minimum productivity are calculated as follows:

$$227 \qquad \Delta d_{dr}^{ab} = d_{dr}^{max} - d_{dr}^{min}. \tag{5}$$

228
$$\Delta d_{dr}^{rel} = \frac{2(d_{dr}^{max} - d_{dr}^{min})}{d_{dr}^{max} + d_{dr}^{min}}.$$
 (6)

To define the optimal number of experiments and the highest accuracy degree and reliability of the obtained results, as well as for the processing of these results, methods of mathematical statistics were used [25].

Two types of measurement errors - random and systematic, may occur during the experimentconducting [26].

A random error reduces the accuracy of experiment results. An analysis of this type of error is
possible by using the root-mean-square deviation σ, calculated by the following equation:

236
$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} \bar{x} - x_{i}^{2}}{n-1}},$$
 (7)

237 where \bar{x} is the arithmetic mean value; x is the single parameter value; n is the number of 238 measurements.

- 239 The maximum possible error of a single measurement, Δ , was determined by the three sigma 240 rule:
- $241 \qquad \Delta = 3\sigma. \tag{8}$

The bilateral confidence interval of the arithmetic mean value ε was determined by the following function [26], provided that this parameter is located in the confidence interval with the probability not less than 95%:

245
$$\varepsilon = t \frac{\sigma}{\sqrt{n}}$$
, (9)

246 where t is the Student's criterion [27].

247 The root-mean-square error of indirect measurements is calculated as:

248
$$\sigma_{\mathbf{y}} = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial \mathbf{y}}{\partial \mathbf{x}} \cdot \Delta \cdot \mathbf{x}_{i}\right)^{2}}$$
, (10)

249 where $y=f(x_1, x_2, ..., x_n)$.

The accuracy of the obtained regression equations is determined by the least-squares method [28].

The systematic measurement error had an identical effect on all parameters that were controlled during the experiment. All measurement devices were calibrated by calibration instruments by comparing their accuracy with declared in the technical documentation in order to exclude the above error. Connection between measurement devices and controllers was provided with a maximum error of processing signals within 1.5%.

257 Creation of graphical dependences was carried out by differential methods of mathematical 258 analysis and integral calculus. Reliability of the obtained experimental results is due to 259 application of time-tested methods in practice.

260 2.3 Materials

Melt of ammonium nitrate and urea was used. Manufacturer of ammonium nitrate
 (agrotechnical chemical) - PSC "Azot", Ukraine. Main parameters of ammonium nitrate –

265

3. RESULTS AND DISCUSSION

The model of the jet decay is based on the solution of Navier-Stokes equations (11) - (12) and the flow continuity equation (13) in cylindrical coordinates [9], with the following simplifications:

269 - flow is axially symmetric;

270 - cross-section of the jet is circular, there is only jet restriction and extension (the tangential 271 component of jet velocity $u_{\theta}=0$).

272
$$\mathbf{v}_{r} \frac{\partial \mathbf{v}_{r}}{\partial \mathbf{r}} = -\frac{1}{\rho} \cdot \frac{\partial \mathbf{p}}{\partial \mathbf{r}} + \mathbf{v} \cdot \left[\frac{\partial^{2} \mathbf{v}_{r}}{\partial z^{2}} + \frac{\partial}{\partial \mathbf{r}} \cdot \left(\frac{\partial}{\partial \mathbf{r}} \mathbf{r} \cdot \mathbf{v}_{r} \right) \right],$$
(11)

273
$$\mathbf{v}_{z} \frac{\partial \mathbf{v}_{z}}{\partial z} = -\frac{1}{\rho} \cdot \frac{\partial \mathbf{p}}{\partial z} + \mathbf{v} \cdot \left[\frac{\partial^{2} \mathbf{v}_{z}}{\partial z^{2}} + \frac{1}{r} \cdot \frac{\partial}{\partial r} \cdot \left(\mathbf{r} \cdot \frac{\partial \mathbf{v}_{z}}{\partial r} \right) \right],$$
(12)

274
$$\frac{\partial v_z}{\partial z} + \frac{1}{r} \cdot \frac{\partial}{\partial r} \cdot r \cdot v_r = 0.$$
 (13)

By assuming that the axial velocity component at the time of leaving the hole variesparabolically with the radial coordinate:

277
$$\upsilon_z = A_1 \cdot r^2 \cdot z^2 + A_2 \cdot r + A_3,$$
 (14)

and transforming the equation (13), we obtain the value of the radial component of the jetvelocity:

280
$$v_r = \frac{-\frac{1}{2} \cdot A_1 \cdot r^4 \cdot z + F_1 z}{r}$$
 (15)

281 where $F_1(z)$ is polynomial function.

282 Given the fact that the pressure change in a jet in the radial direction is insignificant compared

to the axial component, and by substituting (14) into (12) we get:

284
$$2 \cdot A_1 \cdot r^2 \cdot z^2 + A_2 \cdot r + A_3 \quad A_1 \cdot r^2 \cdot z = \frac{1}{\rho} \cdot \frac{dp}{dz} + v \cdot \left(2 \cdot A_1 \cdot r^2 + \frac{4 \cdot A_1 \cdot r \cdot z^2 + A_2}{r}\right).$$
 (16)

By solving the equation (16) for dp/dz and by integration the expression for pressure change along the jet axis is obtained:

287
$$p(z) = -\frac{1}{r} \cdot \left(\rho \cdot \left(\frac{1}{2} \cdot A_1^2 \cdot r^5 \cdot z^4 + A_1 \cdot r^4 \cdot z^2 \cdot A_2 + A_1 \cdot r^3 \cdot z^2 \cdot A_3 - 2 \cdot v \cdot A_1 \cdot r^3 \cdot z - \frac{4}{3} \cdot v \cdot A_1 \cdot r \cdot z^3 - v \cdot A_2 \cdot z \right) \right) + C_1$$
(17)

By setting the origin of the coordinate system at the hole exit (z=0) and by introducing the assumption that the liquid outflow occurs at a constant pressure (p = const), then according to (17) it is obtained that C₁=p₁.

291 After insertion of this constant we get:

292
$$p(z) = -\frac{1}{r} \cdot \left(\rho \cdot \left(\frac{1}{2} \cdot A_1^2 \cdot r^5 \cdot z^4 + A_1 \cdot r^4 \cdot z^2 \cdot A_2 + A_1 \cdot r^3 \cdot z^2 \cdot A_3 - 2 \cdot v \cdot A_1 \cdot r^3 \cdot z - \frac{4}{3} v \cdot A_1 \cdot r \cdot z^3 - v \cdot A_2 \cdot z \right) \right) + p_1(18)$$

By inserting (18) into (11):

294
$$\frac{\left(-\frac{1}{2}\cdot A_{1}\cdot r^{4}\cdot z+F_{1}\cdot z\right)\left(-2\cdot A_{1}\cdot r^{2}\cdot z-\frac{-\frac{1}{2}\cdot A_{1}\cdot r^{4}\cdot z+F_{1}\cdot z}{r^{2}}\right)}{r}=-\frac{1}{\rho}\cdot\left(\frac{1}{2}\cdot A_{1}^{2}\cdot r^{5}\cdot z^{4}+A_{1}\cdot r^{4}\cdot z^{2}\cdot A_{2}+\frac{1}{2}\cdot r^{2}\cdot z^{2}\right)}{r}$$

295
$$+A_1 \cdot r^3 \cdot z^2 \cdot A_3 - 2 \cdot v \cdot A_1 \cdot r^3 \cdot z - \frac{4}{3} \cdot v A_1 \cdot r \cdot z^3 - v \cdot A_2 \cdot z = -\frac{1}{r} \cdot \left(\rho \cdot \left(\frac{5}{2} \cdot r^4 \cdot A_1^2 \cdot z^4 + 4 \cdot A_1 \cdot r^3 \cdot z^2 \cdot A_2 + \frac{1}{3} \cdot v \cdot A_2 \cdot z \right) \right) = -\frac{1}{r} \cdot \left(\rho \cdot \left(\frac{5}{2} \cdot r^4 \cdot A_1^2 \cdot z^4 + 4 \cdot A_1 \cdot r^3 \cdot z^2 \cdot A_2 + \frac{1}{3} \cdot v \cdot A_2 \cdot z \right) \right)$$

296
$$+3\cdot A_{1}\cdot r^{2}\cdot z^{2}\cdot A_{3}-6\cdot v\cdot A_{1}r^{2}\cdot z-\frac{4}{3}\cdot v\cdot A_{1}\cdot z^{3})))+v\cdot \left(\frac{1}{r}\cdot \frac{d^{2}F_{1}\cdot z}{dz^{2}}-4A_{1}\cdot r\cdot z\right)$$
(19)

297 we obtain the differential equation of total derivatives in respect to the function $F_1(z)$. Based on 298 the fact that the derivative du_r/dz is equal to:

299
$$\frac{dv_r}{dz} = \frac{-\frac{1}{2} \cdot A_1 \cdot r^4 \cdot z + \frac{dF_1 z}{dz}}{r},$$
 (20)

300 and the radial velocity component of the jet at z=0 becomes $u_r=0$, we obtain:

301
$$\frac{1}{2}A_1 \cdot r^4 \cdot z = \frac{dF_1 \cdot z}{dz}$$
. (21)

302 By using the boundary conditions $F_1(z=0)=0$ and dF_1/dz (z=0)=0, and put them into the equation

303 (14) we obtain the value of the function F_1 as a polynomial:

304
$$F_{1}(z) = \frac{1}{6} \cdot \frac{(-A_{2} + 8 \cdot A_{1} \cdot r^{3}) \cdot z^{3}}{r} + \frac{1}{48} \cdot \frac{A_{1} \cdot r^{2} \cdot (3 \cdot A_{1} \cdot r^{4} - 12 \cdot A_{2} \cdot r - 8 \cdot A_{3}) \cdot z^{4}}{v}.$$
 (22)

305 Substituting the relation (22) in the equation (15) leads to:

$$306 \qquad v_{r} = \frac{1}{48} \cdot \frac{z \cdot -24 \cdot A_{1} \cdot r^{5} \cdot v - 8 \cdot v \cdot z^{2} \cdot A_{2} + 64 \cdot v \cdot z^{2} \cdot A_{1} \cdot r^{3} + 3 \cdot A_{1}^{2} \cdot r^{7} \cdot z^{3} - 12 \cdot A_{1} \cdot r^{4} \cdot z^{3} \cdot A_{2} - 8 \cdot A_{1} \cdot r^{3} \cdot z^{3} \cdot A_{3}}{v \cdot r^{2}}.$$
(23)

307 The coefficient A_2 can be found by assuming that on the jet surface $r=r_s$ the pressure p is equal

308 to the pressure of the surrounding environment p_0 . This boundary condition can be written as:

$$309 \qquad p_{0} = -\frac{1}{r_{s}} \cdot \left(\rho \cdot \left(\frac{1}{2} \cdot A_{1}^{2} \cdot r_{s}^{5} \cdot z^{4} + A_{1} \cdot r_{s}^{4} \cdot z^{2} \cdot A_{2} + A_{1} \cdot r_{s}^{3} \cdot z^{2} \cdot A_{3} - 2 \cdot v \cdot A_{1} \cdot r_{s}^{3} - \frac{4}{3} \cdot v \cdot A_{1} \cdot r_{s} \cdot z^{3} - v \cdot A_{2} \cdot z \right) \right) + p_{1} \qquad (24)$$

310 Thus, the coefficient A₂ is now calculated as:

$$311 \qquad A_{2} = -\frac{1}{6} \cdot \frac{\left(r_{s} \cdot \left(3 \cdot z^{4} \cdot A_{1}^{2} \cdot \rho \cdot r_{s}^{4} + 6 \cdot z^{2} \cdot A_{1} \cdot \rho \cdot r_{s}^{2} \cdot A_{3} - 12 \cdot z \cdot A_{1} \cdot \rho \cdot v \cdot r_{s}^{2} - 8 \cdot \rho \cdot v \cdot A_{1} \cdot z^{3} - 6 \cdot \rho_{1} + 6 \cdot \rho_{0}\right)\right)}{\rho \cdot z \cdot \left(A_{1} \cdot r_{s}^{4} \cdot z - v\right)}.$$
(25)

312 The coefficient A_3 can be defined by assuming that that if r=0, v_r =0:

313
$$A_{3} = -\frac{1}{6} \cdot \frac{3 \cdot z^{4} \cdot A_{1}^{2} \cdot \rho \cdot r_{s}^{4} + 6 \cdot p_{0} - 12 \cdot z \cdot A_{1} \cdot \rho \cdot v \cdot r_{s}^{2} - 8 \cdot \rho \cdot v \cdot A_{1} \cdot z^{3} - 6 \cdot p_{1}}{z^{2} \cdot r_{s}^{2} \cdot \rho \cdot A_{1}}.$$
 (26)

314 Accordingly,

315
$$\upsilon_{z} = A_{1} \cdot r^{2} \cdot z^{2} - \frac{1}{6} \cdot \frac{3 \cdot z^{4} \cdot A_{1}^{2} \cdot \rho \cdot r_{s}^{4} + 6 \cdot p_{0} - 12 \cdot z \cdot A_{1} \cdot \rho \cdot v \cdot r_{s}^{2} - 8 \cdot \rho \cdot v \cdot A_{1} \cdot z^{3} - 6 \cdot p_{1}}{z^{2} \cdot r_{s}^{2} \cdot \rho \cdot A_{1}}.$$
 (27)

The coefficient A_1 is determined by assuming that at a point close to the origin of the coordinate system $z = z_0$, the exhaust velocity has not yet changed its value and is equal to the flow velocity jet in the hole $U_z = U_{z_0}$ that is:

319
$$\upsilon_{z_{0}} = \frac{1}{6} \cdot \frac{6 \cdot A_{1}^{2} \cdot r^{2} \cdot z_{0}^{4} \cdot r_{s}^{2} - 3 \cdot z_{0}^{4} \cdot A_{1}^{2} \cdot \rho \cdot r_{s}^{4} - 6 \cdot p_{0} - 12 \cdot z_{0} \cdot A_{1} \cdot \rho \cdot v \cdot r_{s}^{2} + 8 \cdot \rho \cdot v \cdot A_{1} \cdot z_{0}^{3} + 6 \cdot p_{1}}{z_{0}^{2} \cdot r_{s}^{2} \cdot \rho \cdot A_{1}}.$$
 (28)

320 When we solve the resulting equation (28) for the coefficient A_1 , we obtain:

$$321 \qquad A_{1} = \frac{1}{3} \cdot \frac{1}{z_{0}^{3} \cdot r_{s}^{2} \cdot \rho \cdot (2 \cdot r^{2} - r_{s}^{2})} (-4 \cdot v \cdot \rho \cdot z_{0}^{2} + 3 \cdot v_{z0} \cdot r_{s}^{2} \cdot \rho \cdot z_{0} - 6 \cdot \rho \cdot v \cdot r_{s}^{2} + (16 \cdot v^{2} \cdot \rho^{2} \cdot z_{0}^{4} - 2 \cdot v_{s}^{2} + 16 \cdot v^{2} \cdot \rho^{2} \cdot z_{0}^{4} - 2 \cdot v_{s}^{2} \cdot \rho \cdot z_{s}^{2} + 16 \cdot v^{2} \cdot \rho^{2} \cdot z_{0}^{4} - 2 \cdot v_{s}^{2} \cdot \rho \cdot z_{s}^{2} + 16 \cdot v^{2} \cdot \rho^{2} \cdot z_{0}^{4} - 2 \cdot v_{s}^{2} \cdot \rho \cdot z_{s}^{2} + 16 \cdot v^{2} \cdot \rho^{2} \cdot z_{0}^{4} - 2 \cdot v_{s}^{2} \cdot \rho \cdot z_{0}^{2} + 12 \cdot v_{s}^{2} \cdot \rho \cdot z_{0}^{2} + 12 \cdot v_{s}^{2} \cdot \rho^{2} \cdot \rho^{2} \cdot z_{0}^{2} + 12 \cdot v_{s}^{2} \cdot \rho^{2} \cdot \rho^{2} \cdot \rho^{2} \cdot z_{0}^{2} + 12 \cdot v_{s}^{2} \cdot \rho^{2} \cdot \rho^$$

$$322 - 24 \cdot \nu \cdot \rho \cdot^2 z_0^3 \cdot \upsilon_{z_0} \cdot r_s^2 + 48 \cdot \nu^2 \cdot \rho^2 \cdot z_0^2 \cdot r_s^2 + 9 \cdot \upsilon_{z_0}^2 \cdot r_s^4 \cdot \rho^2 \cdot z_0^2 - 36 \cdot \upsilon_{z_0} \cdot r_s^4 \cdot \rho^2 \cdot z_0 \cdot \nu + 36 \cdot \rho^2 \cdot \nu^2 \cdot r_s^4 + 6 \cdot \rho^2 \cdot \nu^2 \cdot \rho^2 \cdot \nu^2 \cdot r_s^4 + 6 \cdot \rho^2 \cdot \nu^2 \cdot \rho^2 \cdot \rho^2$$

$$323 \quad 36 \cdot z_0^2 \cdot r^2 \cdot r_s^2 \cdot \rho \cdot \rho_0 - 36 \cdot z_0^2 \cdot r^2 \cdot r_s^2 \cdot \rho \cdot \rho_0 - 36 \cdot z_0^2 \cdot r^2 \cdot r_s^2 \cdot \rho \cdot \rho_1 - 18 \cdot z_0^2 \cdot r_s^4 \cdot \rho \cdot \rho_0 + 18 \cdot z_0^2 \cdot r_s^4 \cdot \rho \cdot \rho_1)^{1/2}) \quad (29)$$

The presented mathematical model allows calculating radial and axial components of the velocity jet outflow, as well as to establish the influence of physical and chemical properties of the liquid and the hole diameter on the jet length and velocity along the axis to its disintegration into separate drops (Figs 2, 3).





- 344 temperature of the melt and the diameter of the perforated shell hole leads to the reduction of
- 345 the distance from the hole, at which the radial component becomes critical, at which the jet is
- 346 broken up. The negative velocity indicates the breakoff of flow (detachment of the jet with the
- 347 formation of the vortex flows), which disappears with an increase of parameter z.
- 348 The smaller the distance from the outflow hole during jet breaking-up, the smaller the length of
- 349 the jet portion, which forms the volume of drop during jet breaking-up. This hypothesis
- 350 coincides with the results of other scientists' studies [30].
- 351 For granulator basket rotation velocity n = 60 rpm and a load of 37 t/h optimal diameter of the
- 352 hole is d=1.2 mm, the melt temperature is 185°C.
- 353 An example of comparison of theoretical calculations and experimental results of velocity radial
- 354 component measurement was shown in Table 1.
- 355
- 356
- 357
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- 364
- 365

366 Table 1. An example of comparison of theoretical calculations and experimental results of v_r

367 measurement (hole diameter was 1.3 mm, temperatures of the melt was 175°C (viscosity was

368 5.36 mPa·s, density was 1434 kg/m³), vibration frequency was 340 Hz, granulator basket

369

rotation velocity was n = 60 rpm and a load was 37 t/h)

z=0 m			<mark>z=0.0005 m</mark>		
<mark>U_{r.theor} ,m/s</mark>	No of	<mark>u_{r.exp} ,m/s</mark>	_{ur.theor} ,m/s	No of	<mark>U_{r.theor},m/s</mark>
	<mark>measurement</mark>			measurement	
	1	0		1	0.04
			-0.05	<mark>1</mark>	-0.04
0	2	-0.003	-0.05	<mark>2</mark>	-0.02
0	<mark>3</mark>	0.002	-0.05	<mark>3</mark>	-0.08
<u>0</u>	<mark>4</mark>	0	-0.05	4	<mark>-0.07</mark>
<mark>0</mark>	<mark>5</mark>	<mark>0.004</mark>	<mark>-0.05</mark>	<mark>5</mark>	<mark>-0.09</mark>
<mark>0</mark>	6	<mark>-0.005</mark>	<mark>-0.05</mark>	<mark>6</mark>	<mark>-0.04</mark>
	<mark>z=0.001 m</mark>			<mark>z=0.0015 m</mark>	
<mark>U_{r.theor} ,m/s</mark>	<mark>No of</mark>	<mark>u_{r.exp} ,m/s</mark>	_{ur.theor} ,m/s	<mark>No of</mark>	<mark>U_{r.theor},m/s</mark>
	<mark>measurement</mark>			<mark>measurement</mark>	
<mark>-0.2</mark>	<mark>1</mark>	<mark>-0,21</mark>	<mark>-0,4</mark>	<mark>1</mark>	<mark>-0,4</mark>
<mark>-0.2</mark>	<mark>2</mark>	<mark>-0,19</mark>	<mark>-0,4</mark>	<mark>2</mark>	<mark>-0,42</mark>
<mark>-0.2</mark>	<mark>3</mark>	<mark>-0,19</mark>	<mark>-0,4</mark>	<mark>3</mark>	<mark>-0,39</mark>
<mark>-0.2</mark>	<mark>4</mark>	<mark>-0,19</mark>	<mark>-0,4</mark>	<mark>4</mark>	<mark>-0,38</mark>
<mark>-0.2</mark>	<mark>5</mark>	<mark>-0,18</mark>	<mark>-0,4</mark>	<mark>5</mark>	<mark>-0,4</mark>
<mark>-0.2</mark>	<mark>6</mark>	<mark>-0,2</mark>	<mark>-0,4</mark>	<mark>6</mark>	<mark>-0,41</mark>
	<mark>z=0.002 m</mark>			<mark>z=0.0025 m</mark>	
<mark>U_{r.theor} ,m/s</mark>	No of	<mark>u_{r.exp} ,m/s</mark>	_{ur.theor} ,m/s	No of	<mark>U_{r.theor},m/s</mark>
	<mark>measurement</mark>			<mark>measurement</mark>	
<mark>0</mark>	<mark>1</mark>	<mark>-0,01</mark>	<mark>2.5</mark>	1	<mark>2,44</mark>
<mark>0</mark>	<mark>2</mark>	<mark>0,01</mark>	<mark>2.5</mark>	<mark>2</mark>	<mark>2,53</mark>
<mark>0</mark>	<mark>3</mark>	<mark>0,02</mark>	<mark>2.5</mark>	<mark>3</mark>	<mark>2,51</mark>
<mark>0</mark>	<mark>4</mark>	<mark>0</mark>	<mark>2.5</mark>	<mark>4</mark>	<mark>2,41</mark>
O	<mark>5</mark>	<mark>-0,03</mark>	<mark>2.5</mark>	5	<mark>2,56</mark>
<mark>0</mark>	<mark>6</mark>	<mark>0</mark>	<mark>2.5</mark>	<mark>6</mark>	<mark>2,6</mark>

Basket tests of the granulator (shown in Fig. 1) confirmed the theoretical research and provided a basis for modernization of the equipment construction. During the tests, a stable jet breakup into drops at a distance of 2-5 mm from the wall of the perforated shell was obtained (Figs 4, 5).



385	
386	Fig. 5. Steady jet disintegration into drops after the outflow from the perforated shell:
387	ammonium nitrate drops, vibration frequency of 340 Hz. Diameter of hole was 1.1 at the
388	temperature of 185°C (viscosity was 4.74 mPa·s, density was 1428 kg/m ³). Granulator basket
389	rotation velocity was n = 60 rpm and a load of 37 t/h.
390	The developed mathematical model was extended with the theoretical description of the melt
391	dispersion process from rotating perforated shells, which allowed us to improve the granulator
392	design to stabilize hydrodynamic parameters of the melt movement within it. By applying of a
393	weighted vortex layer in combination with the vibrating material liquid spray and rotation of
394	liquid jets by their decay will further improve the quality of the granulated product.
395	Scheme of the modernized granulator is shown in Fig. 6, and layout solutions for granulator
396	installation in the granulation tower in Fig. 7. Similarity of respective particles movements and
397	their trajectories in industrial design and in experimental models as maintained.

- Installation of a guiding element in the form of an auger into the granulator, when the melt contacts with the shoulder blade, increases the melt total pressure by transforming the screw mechanical energy into the melt kinetic energy and then turning it into the internal energy. The possibility of screw rotation provides the option for increasing the pressure before the outflow
- 402 holes.





- 408 **14 mesh for the final melt filtration; 15 ring; 16 bolts; 17 pins; 18 cylindrical chamber;**
- 409 **19** hollow shaft; **20** bearing assembly; **21** flange connection; **22** bulge for centering the
- 410 cylindrical chamber; 23 vibration device; 24 rod; 25 disc radiant; 26 hub.
- 411



413

Fig. 7. The layout solutions for granulator installation in the granulation tower

Pilot testing of the modified granulator of the total capacity of **37 t/h** in production of ammonium nitrate for different climatic conditions (humid and hot climate, temperate continental type) showed a higher yield of marketable fractions and reduction of dust content in flue gases.

High level of monodispersity of granules is achieved by improving the fusion hydrodynamics in
the granulator, improving the process of applying vibration to the jets of substance that leak out
from the basket perforated bottom.

Also the modified granulator significantly reduced the granulated product dust level in the air coming out of the tower. Axial flow fans capture from 16 to 38 mg/m³ of dust so that the granulator enables the company to reduce the considerable funds necessary for purchasing expensive new equipment to clean the air coming out of the tower.





427 Fig. 8. Mass fractions of ammonium nitrate granules with sizes 2.0 – 2.5 mm and 2.5 - 3.0 mm

428 as functions of the vibration frequency at the granulator basket rotation velocity of n = 60

429

426

rpm and a load of 37 t/h and the hole diameter of 1.2 mm

430



- 446 granulation process and essentially improves the agrotechnical value of fertilizers.
- 447 Table 2 shows a comparative analysis of the granulometric composition of the final product in
- 448 different types of granulators.
- Table 2. Comparison of rotating vibration granulators with world analogues of the granulation
- 450

equipment in granulation towers

		Acoustics granulator	
	Centrifugal		
Granulometric	granulator	designed by Research	Rotating vibration
Grandionictite	grandiator	Institute at the	granulators (this
composition, %	of firm "Kreber"		C (
		Chemical plant	work)
	(Netherlands) <mark>[31]</mark>	(Russia) <mark>[32]</mark>	
- 1-4 mm	97-99	98-99	more than 99
- 2-4 mm	83-92	85-95	90-97
- 2-3 mm	75-90	80-90	more than 90
- 2.0-2.5 mm	40-50	45-65	more than 80
- less than 1 mm	0.8-2,5	0.8-1.5	0.1-0.8

- 452 The improved granulator has the following advantages over other granulator types (on the basis
- 453 of literature review, e.g. [33-37]):
- 454 high safety in operation;
- 455 production of more competitive uniform granules;
- 456 avoidance of the product's sticking in towers;
- 457 decrease of dust arising;

- increase of the agro-technical value of fertilizers. 458
- 459 The vibrating granulators have a reliable vibration system, which provides a stable imposition of
- 460 oscillations on the fluid jets, flowing out of perforated shell holes, regardless the changes in the
- 461 load on the melt disperser. This vibration system provides measurements of the level of melt in
- 462 the granulator and thereby, to control the clogging degree and the melt outflow velocity from
- 463 the holes of the perforated shell.
- 464 The vibrating granulator (fig. 6) with an electromagnetic vibration system (vibration frequency
- of 340 Hz) provided production of a product with the following granulometric composition as 465
- 466 mass fractions: 0.02-0.2% of granules < 1.0 mm and over 96% of granules 2.0 - 4.0 mm in size.
- 467 Also, for the fraction of granules in the size range 2.0 - 2.5 mm was not lower than 88% with the
- 468 main size in the range 2.1-2.5 mm. When the vibration frequency was changed to 400 Hz, the
- 469 granulator provided production of the product with the main fraction (over 65 %) of granules of
- 470 2.5 - 3.0 mm simultaneously increasing the hardness of the main fraction granules (hardness
- 471 value was confirmed in [12]).
- 472 Similar results of granulometric compositions of products were obtained on vibrating 473 granulators with electromagnetic vibration systems in the ammonium nitrate production under 474 tropical conditions in Cuba and urea with foaming additives of hydrohumates [12]. During the 475 industrial operation of this device the product (urea) with the following granulometric composition in mass fractions was stably obtained during one month: 0.1 - 0.3 % of granules < 476 477 1.0 mm, 99.7 - 99.9 % of granules in the size range 1.0-4.0 mm where granules in the size range 2.0 - 4.0 mm comprised 96.5-98.9% while granules of a size larger than 4.0 mm were absent. 478 479
 - 4. CONCLUSION

480 Consideration of hydrodynamic parameters of the liquid jet flowing out of holes of a perforated 481 membrane allowed us to affect parameters of the process of jet decay into drops, drop size and 482 dispersity and consequently to improve the construction of a nitrogen fertilizers melt 483 granulator.

- 484 Main results:
- 485 the mathematical model to calculate hydrodynamic properties of the melt jet expiration
- 486 process from a perforated shell is formed;
- 487 the influence of the holes diameter and melt properties on the radial velocity field is shown;
- 488 the optimal conditions for prilling in a vibrating granulator for a given capacity of 37 t / h are
- 489 defined: the rotation velocity of the basket, the diameter of the holes in the perforated shell,
- 490 the melt temperature, the frequency range of the actuator's oscillation;
- 491 the modernized construction of the vibrating granulator is given
- 492 results regarding industrial tests of the modernized vibrating granulator are presented, the
- 493 technological features of the work of which complied with the optimal conditions of the prilling.
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- 499 Slovak Republic (KEGA), project No. KEGA 002TnUAD-4/2019 and by the Ministry of Science and
- 500 Education of Ukraine under the project «Small-scale energy-saving modules with the use of

- 501 multifunctional devices with intensive hydrodynamics for the production, modification and
- 502 encapsulation of granules», project No. 0119U100834.
- 503 Symbols
- 504 A_1 , A_2 , A_3 coefficients of parabolic equation;
- 505 C_1 differential equation solution constant;
- 506 d_{dr} diameter of drop, m;
- 507 d_{dr}^{max} , d_{dr}^{min} bigger and smaller diameter of drops, m (mm);
- 508 Δd_{dr}^{ab} absolute difference of drop diameters, m;
- 509 Δd_{dr}^{rel} relative deviation of drop diameters;
- 510 $F_1(z)$ polynomial function;
- 511 f_1, f_2 upper and lower limits of frequency, 1/s;
- 512 f_{avi} average frequency of vibration, which provides monodisperse jets decay, 1/s;
- 513 g acceleration of gravity, m/s^2 ;
- 514 G_s melt flowrate, m³/s;
- 515 G_{τ} the measured volume of the melt, m³;
- 516 H liquid column height (head), m;
- 517 n number of measurements; granulator basket rotation velocity, rpm;
- 518 p the outflow jet pressure, Pa;
- 519 $p_0 pressure of the surrounding environment, Pa;$
- 520 r radius of the jet, m;
- 521 t the Student's criterion;
- 522 V velocity of the liquid leaking from the granulator holes, m/s;

- \overline{x} arithmetic mean value;
- 524 x single parameter value;
- $z_0 initial axial distance, holes, m;$
- 526 z axial distance, holes, m;
- Δ maximum possible error of a single measurement, %;
- ϵ bilateral confidence interval of the arithmetic mean value;
- λ_{max} , λ_{min} maximum and minimum lengths of the wave, m;
- ρ liquid density, kg/m³;
- 531 σ root-mean-square deviation;
- τ the experimental time of melt outflow through the granulator hole, s.
- $v_{\theta} v_{r}$, v_{z} tangential, radial and axial components of the jet velocity respectively, m/s;
- U_{z_0} initial axial component of the jet velocity respectively, m/s;
- ϕ discharge coefficient.

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