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ADVANCED pH NEUTRALIZATION CONTROL USING MODEL REFERENCE ADAPTIVE CONTROL (MRAC) WITH MIT RULE

Highlights

- Development of MRAC-based control strategy with MIT rule
- Impact of adaptation gain
- Practical insights for process control
- Comparison with conventional methods: PID vs MRAC
- Theoretical and experimental validation in neutralization process

Abstract

This study presents the design and implementation of a Model Reference Adaptive Controller (MRAC) using the Massachusetts Institute of Technology (MIT) rule for a pH neutralization process in a continuous reactor. The inherent nonlinearity of acid-base reactions makes conventional Proportional-Integral-Derivative (PID) control insufficient in handling rapid pH variations. To address this, an adaptive control strategy was proposed, allowing the system to dynamically adjust control parameters based on real-time deviations from the reference model. The adaptation gain (γ) played a critical role in system stability and performance, with simulations and experimental results confirming that γ = 0.025 yielded optimal response characteristics. Higher adaptation gains accelerated convergence but introduced oscillations, while lower values slowed the response. MATLAB/Simulink simulations and real-time experimental validation demonstrated that MRAC effectively stabilized the system, achieving faster settling time and improved tracking performance compared to PID control. The findings suggest that MRAC with the MIT rule is a viable alternative for complex nonlinear processes, offering improved robustness against disturbances and set-point variations. Further enhancements, including the Normalized MIT rule and polynomial modeling, could further refine the controller's effectiveness in industrial applications.

Keywords: model reference adaptive control (MRAC), MIT rule, adaptation gain, PID control, nonlinear systems, pH neutralization.

INTRODUCTION

The regulation of pH levels is a critical aspect of pharmaceutical, chemical, and biotechnological engineering, with wastewater neutralization being one of the most extensively studied cases [1]. Ensuring that wastewater effluent maintains a pH value within the safe range of 6-8 before discharge is essential for environmental safety and regulatory compliance [2]. However, achieving precise pH control is challenging due to its inherent nonlinearity. The system becomes highly sensitive near the neutralization point, and variations in flow rates and concentrations of neutralization agents introduce further complexity. Major difficulties include the sensitivity of the titration curve, nonlinearity, buffer selection, volume/agitation configurations, and dead time. Overcoming these challenges requires a strong foundation in control theory and meticulous system design [2].

Numerous pH control strategies have been developed to address nonlinearity, disturbances, steady-state/transient characteristics, setpoint tracking, robustness, and time-varying behaviors [2]. Despite extensive research, pH regulation remains a topic of ongoing investigation due to its complexity. Traditional linear control techniques, such as Proportional-Integral-Derivative (PID) and Proportional-Integral (PI) controllers, are widely used and often combined with feedforward control for predictive action [3]. However, these conventional approaches struggle with nonlinearity and lack adaptability to dynamic process variations.

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To address these limitations, modern control strategies such as adaptive, model-based, and self-tuning control systems have been explored. Adaptive control, initially proposed by Åström [4] and Seborg *et al.* [5], is particularly suited for dynamic environments, as it automatically adjusts controller settings in response to system variations. There are two main forms of adaptive control: direct and indirect. Indirect adaptive control estimates system parameters and adjusts the controller accordingly, whereas direct adaptive control, such as Model Reference Adaptive Control (MRAC), modifies controller parameters in real time without explicit parameter estimation [6].

MRAC is an advanced adaptive control strategy designed for improved performance and robustness. It continuously adjusts controller parameters to ensure the system output follows a predefined reference model, making it more effective than PID controllers in handling disturbances and environmental changes [7]. MRAC also maintains system stability despite unforeseen variations, modifying control coefficients dynamically to address disturbances [8]. Adaptive controllers operate through two loops: a classical feedback loop and a parameter adjustment loop. While the classical feedback loop corrects errors between the system output and the reference model, the parameter adjustment loop updates controller settings to maintain desired performance (Figure S-1) [9]. Given its complexity, MRAC is often implemented using digital computers. The method has been further enhanced through fuzzy logic [10] and various adaptation mechanisms, including the Massachusetts Institute of Technology (MIT) rule [11], Lyapunov theory [8], and the theory of augmented error [12] techniques.

The Lyapunov stability theory is used in Lyapunov-based adaptive control, a control strategy, to guarantee system stability even in the face of unknown parameters. An adaptive control law that modifies the control inputs by estimation of the unknown parameters is derived using the Lyapunov-based adaptive control ensures stability by deriving control laws based on Lyapunov functions, allowing adaptation to unknown or time-varying parameters. However, selecting an appropriate Lyapunov function can be challenging, and the method is sensitive to noise and model inaccuracies, potentially leading to suboptimal performance [8].

MIT rule-based adaptive control, in contrast, adjusts control inputs based on a predefined adaptation rule that depends on the error magnitude and sign. It offers simplicity and ease of implementation but is primarily suited for low-order models and systems with simple dynamics. Additionally, measurement noise and estimation errors can degrade its performance [13]. Despite their similarities, MIT and Lyapunov-based approaches differ in their adaptation mechanisms, with MIT offering a more intuitive approach to adaptation gain selection [14].

Given these considerations, this study proposes an MIT-based Model Reference Adaptive Control (MIT-based MRAC) strategy to address the strong nonlinearity of pH control, the sensitivity of the titration curve, and the safety and regulatory requirements of industrial-scale neutralization processes. The effectiveness of the proposed strategy was validated through theoretical analysis and experimental implementation, demonstrating promising results.

Model Reference Adaptive Control (MRAC)

The MRAC technique utilizes a reference model to define the desired system performance, which is continuously evaluated using input and output data. An adaptation rule is employed to integrate the measured data with the reference model's output, enabling adjustments to the controller and ensuring that the system output aligns with the model output [15].

As illustrated in Figure S-2, the MRAC framework consists of two feedback loops: one regulating the process and controller, while the other modifies controller parameters. The system error is determined by computing the difference between the reference model output and the actual system output. Based on this error, the controller parameters are adjusted accordingly. In Eq. 1 (Figure 1),

$$e(t) = y(t) - y_m(t) \quad (1)$$

where $e(t)$ represents the deviation between the reference and actual process output, while $y(t)$ and $y_m(t)$ denote the actual system output and reference model output, respectively [16].

The control law modifies system parameters based on Eq. 1, while the adaptation law identifies key response parameters that align with the reference model. This design ensures system stability and minimizes tracking errors. Mathematical approaches such as the MIT rule, Lyapunov theory, and augmented error theory enhance the adaptation mechanism, improving its accuracy. By continuously adjusting the controller parameters, the system output is guided to follow the reference model [9]. In this study, the MIT rule was implemented to optimize the tuning mechanism and enhance control performance.

MIT Rule for MRAC

The MIT rule is the foundational approach to MRAC. Originally developed at MIT's Instrumentation Laboratory for aerospace applications using analog hardware, this method became widely known as the MIT rule. It is a versatile technique that can be utilized to design controllers within an MRAC framework for various systems[15].

In this study, the process under investigation involves the neutralization of a strong base (NaOH) with a strong acid effluent (HCl) in a continuous reactor. The procedure is modeled using a first-order dynamic system with nonlinearity, represented through mass balance equations. This modeling approach was initially introduced by McAvoy *et al.* [17] and has since been employed in several recent studies, including those by Gupta *et al.* [18].

$$\frac{dy}{dt} = -ay + bu \quad (2a)$$

where y is the measured output, and u is the control variable, a and b are the parameters of the reference model determining the system dynamics, which are unknown during the control design procedure. We want to obtain a closed-loop system described by:

$$\frac{dy_m}{dt} = -a_m y_m + b_m u_c \quad (2b)$$

where y_m is the reference model output, a_m and b_m are the reference model parameters defining the desired closed-loop dynamics, and u_c is the reference input signal to the reference model.

The purpose of MRAC is to calculate the control input that minimizes the difference between the process output and the reference model output.

The controller can be given as follows,

$$u(t) = \theta_1 u_c(t) - \theta_2 y(t) \tag{3}$$

where θ_1(t) and θ_2(t) are the parameters determined by an adaptive control algorithm, u_c(t) is the reference input function, and y(t) is the system output.

Substitute control law Eq. (3) in Eq. (2a),

$$\frac{dy}{dt} = -(a + b\theta_2)y + b\theta_1 u_c \tag{4}$$

Eq. (4) gives a closed-loop system. If Eq. (4) is compared with Eq. 2, and the parameters θ_1 and θ_2 are chosen as

$$\theta_1 = \theta_1^o = \frac{b_m}{b}$$

and

$$\theta_2 = \theta_2^o = \frac{a_m - a}{b} \tag{5}$$

where θ_1^o and θ_2^o are the ideal values of adaptive controller parameters, a and b are unknown plant parameters appearing in the first-order process model.

If the input-output relations of the system and the reference model are the same, this is called perfect model-following [15]. So, it can be concluded that the reference model (Eq. (2b)) and the closed-loop system's dynamic behaviour (Eq. (4)) are identical. However, in practice, the constants a and b of the system are not known; therefore, the parameters θ_1 and θ_2 cannot be determined in this way. However, the analysis presented above will assist in confirming the adaptive controller convergence that will be achieved.

The purpose of the MIT adaptive control rule is to minimize the cost function (J) [15]:

$$J = \frac{1}{2} e^2 \tag{6}$$

where e is the error given in Eq. (1). Minimizing J can be done by changing the parameters θ_1 and θ_2 in the direction of the negative gradient of J as:

$$\frac{d\theta_1}{dt} = -\gamma e \frac{\partial e}{\partial \theta_1} \tag{7a}$$

$$\frac{d\theta_2}{dt} = -\gamma e \frac{\partial e}{\partial \theta_2} \tag{7b}$$

where $\frac{d\theta_1}{dt}$ and $\frac{d\theta_2}{dt}$ are first derivatives of adaptive parameters. γ is the adaptation gain governing the speed of parameter adjustment.

This is the MIT rule[15]. The sensitivity derivative, the partial derivative of the system ∂e/∂θ, shows how the error is affected by adjustable parameters. When the parameter variations are not as fast as the other variables in the system, then the derivative ∂e/∂θ, θ can be assumed as constant. To compute the partial derivatives, y(t) needs to be expressed from Eq. (4). The differential operator p can be used for this purpose as:

$$pf(t) = \frac{df(t)}{dt} \tag{8}$$

where p denotes the differential operator.

Consequently, from Eq. (4), we obtain:

$$y = \frac{b\theta_1 u_c}{p + a + b\theta_2} \tag{9}$$

where θ_1 is the adaptive control parameter, u_c is the control input, a and b are the plant parameters, θ_2 is the second adaptive parameter, and p denotes the differential operator.

If the error is written using Eq. (9):

$$e = \frac{b\theta_1 u_c}{p + a + b\theta_2} - y_m \tag{10}$$

where e(t) is the tracking error, and y_m is the reference model output.

By taking partial derivatives, θ_1 and θ_2, the sensitivity derivatives are obtained as:

$$\frac{\partial e}{\partial \theta_1} = \frac{bu_c}{p + a + b\theta_2} \tag{11a}$$

$$\frac{\partial e}{\partial \theta_2} = \frac{-b^2\theta_1}{(p + a + b\theta_2)^2} u_c = \frac{-by}{p + a + b\theta_2} \tag{11b}$$

where θ_1 is the adaptive control parameter, u_c is the control input, a and b are the plant parameters, and θ_2 is the second adaptive parameter. The partial derivatives in Eq. (11a) and (11b), depending on the unknown system parameters a and b, cannot be calculated in practice as mentioned before; therefore, those equations cannot be applied directly [15].

Fundamentally, in the ideal situation, the values θ_1 and θ_2 should converge to the parameters θ_1 and θ_2, which equates closed-loop dynamics equal to reference model dynamics. In this ideal case, from Eq. (5):

$$a_m = a + b\theta_2^o \cong a + b\theta_2 \tag{12}$$

where a_m is the parameter of the reference model.

Accordingly, the unknown terms in Eq. (11a) and (11b) can be assumed as:

$$p + a + b\theta_2 \cong p + a_m \tag{13}$$

where p denotes the differential operator, a and b are the plant parameters, θ_2 is the adaptive feedback parameter, and a_m is the reference-model pole, which will be logical when parameters are close to their correct values. Next, substituting Eq. (13) into Eq. (11):

$$\frac{\partial e}{\partial \theta_1} \cong \frac{bu_c}{p + a_m} \tag{14a}$$

$$\frac{\partial e}{\partial \theta_2} \cong \frac{-by}{p + a_m} \tag{14b}$$

where $\frac{\partial e}{\partial \theta_1}$ and $\frac{\partial e}{\partial \theta_2}$ are partial derivatives.

Finally, by substituting these partial derivatives in Eq. (7), parameters change in time can be achieved.

$$\frac{d\theta_1}{dt} = -\gamma \frac{a_m u_c}{p + a_m} e \tag{15a}$$

$$\frac{d\theta_2}{dt} = \gamma \frac{a_m y}{p + a_m} e \tag{15b}$$

where $\gamma = \frac{\alpha b}{a_m}$ is the gain.

Dynamics of the closed loop system is described by Eq. (15) and Eq. (9) with the forms of the differential operator p and mixed time-domain. For further enhancement, it is informative to give these equations in pure time-domain and by conducting a few algebraic steps (Eq. (16)),

$$\left.\begin{aligned} &\ddot{\theta}_1 + a_m\dot{\theta}_1 = -\gamma a_m u_c y + \gamma a_m y_m u_c \\ &\ddot{\theta}_2 + a_m\dot{\theta}_2 = \gamma a_m y^2 - \gamma a_m y_m y \\ &\dot{y} = -ay + b\theta_1 u_c - b\theta_2 y \\ &\dot{\theta}_i = \frac{d\theta_i}{dt}\ ;\ \ddot{\theta}_i = \frac{d\dot{\theta}_i}{dt} \end{aligned}\right\} \tag{16}$$

where $\ddot{\theta}_1$ and $\ddot{\theta}_2$ are the second time derivative of adaptive parameters.

The adaptation gain (γ) is a crucial tuning parameter that determines the rate at which the controller adapts. Its primary objective is to strike a balance between fast convergence and system stability. The adaptation gain significantly influences the behavior of the adaptation rule. While a higher adaptation gain accelerates convergence and parameter adjustments, it may also lead to system instability and overshooting. On the other hand, a lower adaptation gain enhances stability but may result in slower convergence and reduced tracking performance. The selection of an appropriate adaptation gain depends on the required performance criteria and the characteristics of the controlled system. Various methods, such as trial-and-error, optimization algorithms, or advanced control design techniques, can be employed to determine the optimal adaptation gain [18,19,20].

The controller's performance is evaluated based on parameters such as peak time, peak overshoot, settling time, integral square error (ISE), integral of absolute error (IAE), integral of time-absolute error (ITAE), integral of time-square error (ITSE), and steady-state error (SSE).

To tune the adaptation gain γ systematically, two global optimization methods were employed: Genetic Algorithm (GA) and Particle Swarm Optimization (PSO).

Material and Method

The neutralization process was conducted in a continuous reactor with a working volume of 2 L, as illustrated in Figure 1(a). The reactor had two inlet streams: an acidic solution of 0.014 M HCl and an alkaline solution of 0.022 M NaOH. The effluent was continuously removed through a drainage pipe. The base flow rate served as the manipulated variable, while the acid flow rate was maintained at a constant 15 mL/min during neutralization. In both simulation and experiments, base flow rate was constrained within [0, 20 mL/s] to reflect pump capacity and prevent actuator saturation. These constraints were enforced in MATLAB/Simulink models and in the experimental setup. To introduce disturbances, the acid flow rate was increased to 30 mL/min for a duration of 60 seconds. The pH of the solution was continuously monitored using a glass-electrode pH meter connected to a data acquisition system. A mechanical stirrer (IKA-Werke) operated at 300 rpm to ensure thorough mixing throughout the process. Communication was established using MATLAB software, which was installed on a Personal Computer. pH control for pH neutralization was done with the MATLAB Simulink program by using MRA.

The real-time closed-loop control system consisted of a Peri-star peristaltic pump for base addition, a WTW pH 340i pH electrode connected to an EMAF PH199 pH meter, ICP CON 7017F, and ICP CON 7024 as input and output modules, respectively. Super Logics 8520 was used as a data acquisition system, RS 232 to RS-485. Control algorithms were implemented in MATLAB/Simulink. The computer was an Intel® Core™ i7, 16 GB RAM, Windows 7 Ultimate system.

The overall system structure, MATLAB-Simulink diagram used in theoretical and experimental studies for MIT-based MRAC control, was given in Figure 1(b), where the manipulated variable is the base flow rate, the controlled variable is the reactor pH, and the acid flow rate acts as a disturbance. This diagram, combined with the nonlinear mass-balance model, provides the basis for both simulation and experimental studies.

The transfer function model of the system was obtained from open open-loop response as follows:

$$G_P(s) = \frac{0.33}{0.66s+1} \quad (17)$$

where G_p is the first-order transfer function of the system.

RESULTS AND DISCUSSION

Simulation Results

The reference model, represented by Eq. (2b), can be simulated for a selected manipulated variable $u_c(t)$. This allows the reference model output y_m(t), which is a known function of time, to be obtained. The time series {$u_c(t)$,y_m(t)} serves as predefined inputs to guide the system dynamics. The reference model output y_m(t) was compared with the actual system output y(t) to evaluate performance.

The system was simulated using different adaptation gain values (γ = 0.005, 0.01, 0.025, and 0.25), and the results are presented in Figures 2-3. Each figure consists of three sections: (a) illustrates the reference input, reference model output, and the controlled output signal (pH); (b) depicts the variations in the manipulated variable, specifically the base flow rate (u); and (c) displays the convergence trajectories of the controller parameters θ_1 and θ_2 for different γ values. To analyze the system's regulatory behavior, a disturbance was introduced after each setpoint change, effectively simulating a load effect. As observed in the simulations (Figs. 2 and 3), the acid flow rate, acting as the disturbance, was increased from 15 mL/min to 30 mL/min for 60 s at pH values of 7 (disturbance 1 at 2500 s), 8 (disturbance 2 at 5500 s), 9 (disturbance 3 at 8500 s), and 10 (disturbance 4 at 10500 s).

The simulation results indicate that increasing the adaptation gain (γ) reduced the difference between the process output and reference model output after the initial transient phase. However, higher γ values also introduced oscillations during control transients. The oscillation amplitude increased with larger γ values, while at very low γ values, the oscillation period was prolonged. Conversely, at extremely high γ values, oscillations became more frequent. This behavior is evident in Figure 2, where oscillations are prominent during the initial 1600 s.

Increasing the adaptation gain improved system performance, with the chosen γ range set between 0.005 and 0.25. A higher γ resulted in a reduction in settling time but also led to an increase in peak overshoot. It was observed that system performance improved significantly with increasing γ up to a value of 0.025, beyond which excessive oscillations occurred. At γ = 0.025, the best results were achieved, with controller parameters θ_1 and θ_2 converging to 42 and 38, respectively, as shown in Figure 3.

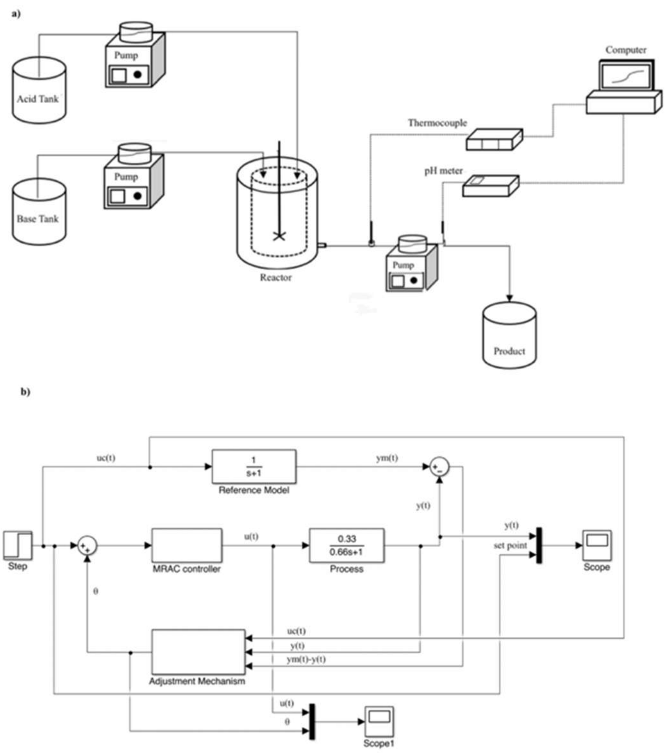

Figure 1. a) Experimental set-up, b) Simulink diagram of the Model Reference Adaptive Controller (MRAC) with MIT rule, showing the process input (base flow rate), controlled output (pH), and disturbance (acid flow rate).

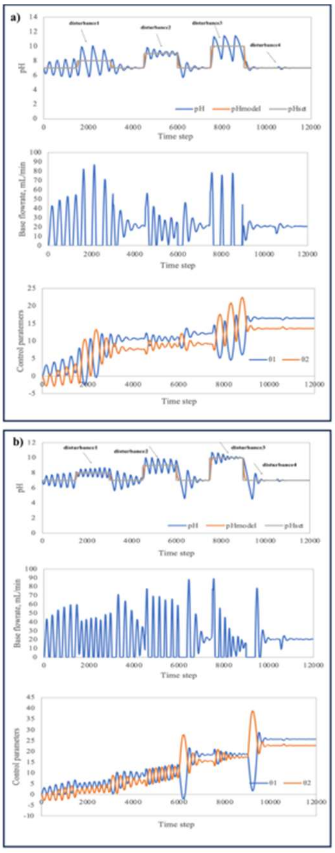

Figure 2. Simulation result of MRAC with MIT rule regulatory control problem with variation of control parameters θ_1 and θ_2 with time (the acid flow rate which is the load effect (disturbance) was increased twice (15 mL/min to 30 mL/min) for 60 seconds at pH values of 7 (disturbance 1 at 2500 s), 8 (disturbance 2 at 5500 s), 9 (disturbance 3 at 8500 s), 10 (disturbance 4 at 10500 s). The upper subplot shows the controlled variable (pH response), while the lower subplot shows the manipulated variable (base flow rate) and adaptive parameters: a) γ = 0.005 and b) γ = 0.010.

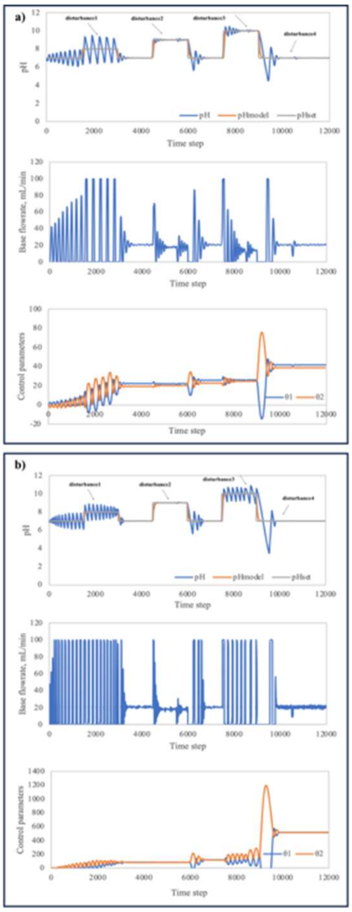

Figure 3. Simulation result of MRAC with MIT rule regulatory control problem with variation of control parameters θ_1 and θ_2 with time (the acid flow rate which is the load effect (disturbance) was increased twice (15 mL/min to 30 mL/min) for 60 s at pH values of 7 (disturbance 1 at 2500 s), 8 (disturbance 2 at 5500 s), 9 (disturbance 3 at 8500 s), 10 (disturbance 4 at 10500 s)). The upper subplot shows the controlled variable (pH response), while the lower subplot shows the manipulated variable (base flow rate) and adaptive parameters: a) γ = 0.025 and b) γ = 0.25.

Table 1 summarizes the performance of the MRAC controller under different adaptation gains (γ), evaluated through standard error indices including ISE, IAE, ITAE, ITSE, and SSE. The results clearly demonstrate the trade-off between responsiveness and stability in the MRAC design.

For very small adaptation gains (γ = 0.005 and 0.010), both the simulation and experimental indices show large IAE and ITAE values, indicating sluggish response with poor tracking of the reference trajectory. Although these low γ values avoid oscillations, the resulting controllers adapt too slowly, leading to high accumulated error.

At the other extreme, a large adaptation gain (γ = 0.25) produces the highest ITSE and SSE values, reflecting oscillatory and unstable behavior. This confirms that excessive adaptation speed destabilizes the system, consistent with known limitations of the MIT rule.

Intermediate values (γ = 0.050) achieve faster convergence compared to small gains but still suffer from larger error integrals than the optimal case, showing a balance tilted slightly toward instability.

The minimum error metrics across all indices occur consistently at $\gamma \approx 0.025$. Both simulation and experimental results confirm that this value yields the lowest ISE, IAE, ITAE, ITSE, and SSE, corresponding to accurate tracking, fast settling, and minimal steady-state error.

Therefore, γ = 0.025 is identified as the most suitable adaptation gain, providing the best trade-off between convergence speed, robustness, and stability for the pH neutralization process.

To validate the empirical tuning, GA and PSO were applied with ITAE as the objective function. In GA, a population size of 20, a crossover probability of 0.8, and a mutation probability of 0.05 were used over 50 generations. For PSO, the swarm size was 30 with an inertia weight of 0.7 and cognitive/social coefficients set to 1.5. Both methods converged to γ values essentially identical to the trial-and-error optimum (0.025). GA yielded γ = 0.0248, and PSO yielded γ = 0.0251 in simulations, while both converged to γ = 0.0250 in experiments. The results confirmed that the empirically observed best value (γ = 0.025) is indeed optimal according to ITAE minimization. Thus, optimization-based tuning validates the manual findings while ensuring reproducibility and robustness. This demonstrates that the chosen adaptation gain is indeed optimal according to formal optimization, confirming the reproducibility and robustness of the tuning process. This also shows that the selected γ is not only supported by experimental observation but also by systematic optimization, ensuring reproducibility and robustness.

Experimental Results

Experimental studies were carried out using PID and

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MRAC controllers to compare and find the most suitable controller implementation for the neutralization system.

PID Controller

PID control is the widely used control technique in industrial processes regarding stability, easy implementation, ease of use, and robustness. Contrary to positive effects, the major disadvantage of the PID is the feedback algorithm, where no control action can be seen until a disturbance occurs and the error is calculated, as well as a long recovery time depending on the dynamics of the process, resulting in poor performance.

Dynamic analysis was applied to find the transfer function of the system, and PID control parameters KC, τ_I, and τ_D were calculated as 10, 100 s^{-1}, and 1 s, respectively.

The acid flow rate, which is the load effect, was increased twice for 60 seconds at pH values of 7 (disturbance1 at 2500 s), 8 (disturbance2 at 5500 s), 9 (disturbance3 at 8500 s), and 10 (disturbance4 at 10500 s); the ISE value for experimental PID was 2051.3. As can be seen in Figure 4, PID control was successful on set point trajectories, but oscillations occurred.

Table 1. Comparison of system responses at different values of adaptation gain for simulation and experimental.

	Adaptation gain (γ)	Peak time (s)	Peak overshoot (pH)	Settling time (s)	ISE	IAE	ITAE	ITSE	SSE
Simulation	0.005	243.5	7.85	4347	90	13	330	85	0.38
	0.01	182	7.65	7332	65	10.8	270	62	0.24
	0.025	150	7	3565	24	5.2	122	26	0.02
	0.25	195	7	4020	120	15.8	410	115	0.42
Experimental	0.005	250	9.1	2500	98	14.6	360	94	0.42
	0.01	238	8.72	2200	71	12.1	295	68	0.27
	0.025	93	8.36	1089	27	5.6	128	29	0.03
	0.25	209	8.9	NAN	130	16.5	435	125	0.45

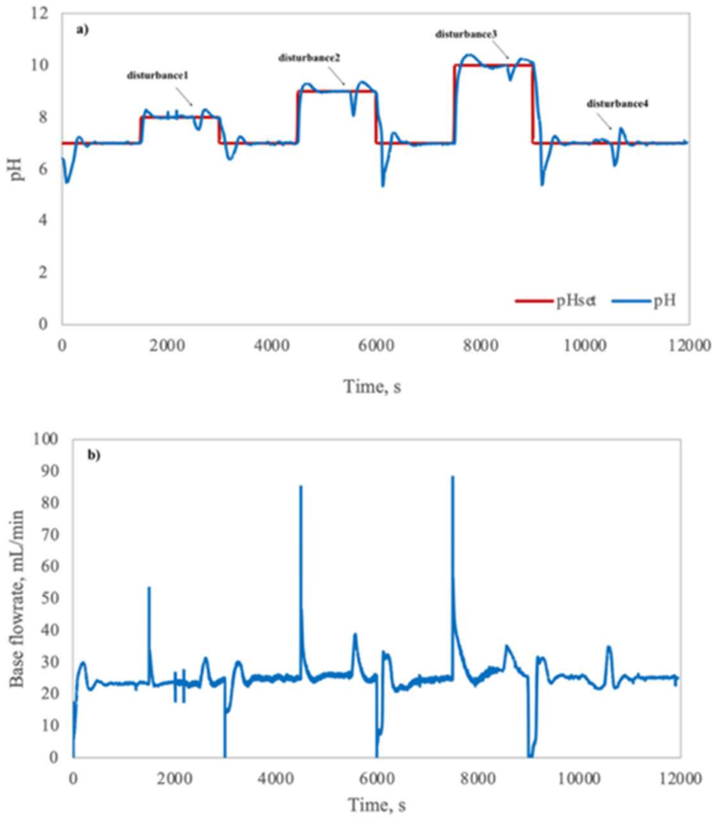

Figure 4. Experimental results for conventional PID control (Kc =10, $\tau_I = 100$, $\tau_D = 1$): a) time response curve b) manipulated variable curve (the acid flow rate which is the load effect (disturbance) was increased twice (15 mL/min to 30 mL/min) for 60 seconds at pH values of 7 (disturbance 1 at 2500 s), 8 (disturbance 2 at 5500 s), 9 (disturbance 3 at 8500 s), 10 (disturbance 4 at 10500 s). The upper subplot shows the controlled variable (pH response), while the lower subplot shows the manipulated variable (base flow rate).

Model Reference Adaptive Control

The primary objective of MRAC is to determine the control input that ensures the system output closely follows the reference model output. The reference model is simulated to generate the desired reference output. MRAC is one of the most commonly used adaptive control methodologies due to its strong tracking performance, flexibility, and minimal constraints [22]. However, if the system model is poorly defined or an incorrect reference model is chosen, it may lead to instability in the control system.

Mathew *et al.* [23] implemented MRAC using the MIT rule to regulate temperature in a dual-tank continuous stirred tank reactor (CSTR), working with adaptation gains ranging from -0.5 to 2 and achieving favorable results [23]. Similarly, Luo *et al.* [24] demonstrated that MRAC simulations on microbial fuel cells delivered rapid and precise control performance [24]. Singh *et al.* [25] investigated the application of MRAC based on power factor under different operating conditions for a solar-powered drive-based water pumping system [25].

In this study, the gamma values tested in the simulations were also validated experimentally, as shown in Figures 5 and 6. The experimental results closely aligned with the simulation findings, indicating consistency between both approaches. Figures 5 and 6 illustrate that increasing gamma values led to a reduction in error. However, higher gamma values also induced oscillations in the transient response. As the gamma value increased, the amplitude of these oscillations grew, whereas at lower gamma values, the oscillation period was extended.

To examine the system's response to disturbances, the acid flow rate, acting as a load effect, was increased twice for 60 seconds at pH values of 7 (disturbance 1 at 2500 s), 8 (disturbance 2 at 5500 s), 9 (disturbance 3 at 8500 s), and 10 (disturbance 4 at 10500 s). The reference model quickly adapted to the setpoint, and the process output stabilized at 1089 s in Figure 6. The manipulated variable responded as expected, and the pH was effectively regulated using the MRAC controller.

The initial values of MRAC parameters θ_1 and θ_2 play a crucial role in control performance. If specific values are unknown, these parameters can be initialized to zero. In this study, all experiments used zero as the initial parameter values. If MRAC control is initiated near the setpoint of both the process output and model output, overshoots and undershoots are inevitable during adaptation. This behavior was observed in both the experimental and simulation results. Setting θ_1 and θ_2 to non-zero values or using the converged parameters from previous control experiments as initial values could improve control performance. Reducing oscillations may also be achieved by initializing θ_1 and θ_2 appropriately or by employing uncertainty-handling techniques [26]. Additional ly, the normalized MIT rule can be applied to minimize oscillations caused by setpoint variations [15].

In this study, θ_1 and θ_2 were initialized at zero for simplicity, which allowed the controller to adapt from a neutral starting point without bias. It should be noted, however, that initialization can influence the early transient response: non-zero values closer to the true system parameters may reduce initial overshoot and shorten adaptation time. While not explored here, systematic selection of initial estimates represents a potential avenue for improving transient performance in future work.

During experimental studies, both positive and negative changes were applied to the setpoint. As shown in Figures 5-6, when a positive change was introduced, the process output successfully followed the model output. However, when a negative change occurred, the process output struggled to match the model output, indicating that the acid effect was slower compared to the base effect. To mitigate such discrepancies, the adaptive algorithm continuously updates the parameters. This adjustment process can cause the parameters–initially tuned to nominal values in response to positive variations–to deviate from their optimal values when negative setpoint changes occur. To prevent this parameter drift, two different models can be utilized. Additionally, a polynomial model incorporating a step response towards the acidic side, based on pH behavior and model output dynamics, can further improve system performance.

The reference model was chosen as a first-order nonlinear structure to capture the dominant dynamics of the pH neutralization process. Step-response experiments indicated that the process exhibits a single dominant time constant with a nonlinear gain variation, which is consistent with previous studies on pH control systems. Model validation against experimental data showed that the first-order nonlinear representation achieved a high degree of fit ($R^2 \approx 0.90$), confirming that it adequately describes the main input-output dynamics for MRAC design.

In Figures 2-6, the lower subplots display the manipulated variable (base flow rate) trajectories for both PID and MRAC. It can be seen that MRAC produces slightly larger transient control actions, particularly during disturbance rejection, but stabilizes more quickly and requires less sustained effort than PID. This demonstrates that MRAC achieves superior performance with only a modest increase in transient control activity.

The performance values obtained in experimental studies were also given in Table 1. As can be seen from ISE, peak time, peak overshoot, and settling time, 0.025 gamma was found to be the best value for controlling pH with MRAC.

Upon analyzing the ISE values, it was found that the simulation results closely matched the experimental data. As anticipated, the ISE value for the PID controller was lower than that of the MRAC controller. However, in the case of an adaptive control algorithm, where no prior knowledge of the process is available at the initial stage, a more appropriate comparison would involve calculating the ISE values only after the control parameters have reached an approximately steady-state condition.

It should be noted that the MIT rule, while attractive for its simplicity and low computational cost, does not inherently guarantee stability. By contrast, Lyapunov-based MRAC formulations ensure global stability through rigorous energy function analysis. In this study, our focus was on demonstrating the feasibility and experimental performance of the MIT rule for pH neutralization control, given its practicality for real-time applications. Nevertheless, future work will consider extending the framework toward Lyapunov-stable adaptive control to combine guaranteed stability with the demonstrated experimental feasibility of the MIT rule.

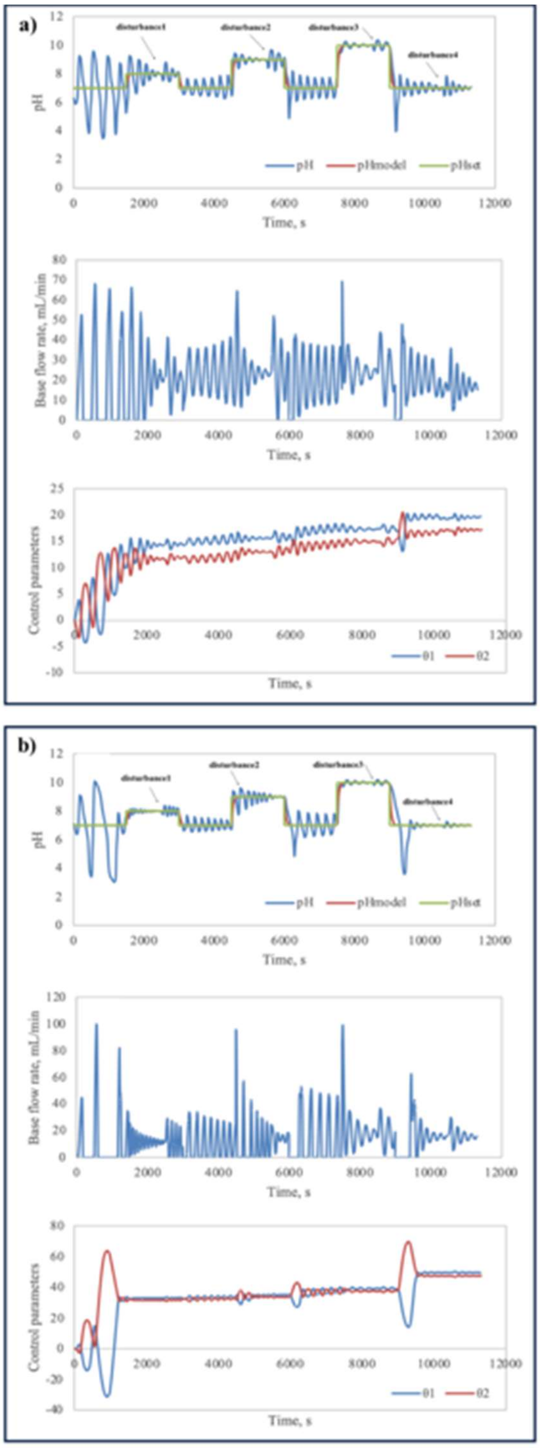

Figure 5. Experimental results for Model Reference Adaptive Control (MRAC) with variation of control parameters θ_1 and θ_2 with time (the acid flow rate which is the load effect (disturbance) was increased twice (15 mL/min to 30 mL/min) for 60 s at pH values of 7 (disturbance1 at 2500 s), 8 (disturbance 2 at 5500 s), 9 (disturbance 3 at 8500 s), 10 (disturbance 4 at 10500 s). The upper subplot shows the controlled variable (pH response), while the lower subplot shows the manipulated variable (base flow rate) and adaptive parameters: a) γ = 0.005 and b) γ = 0.01.

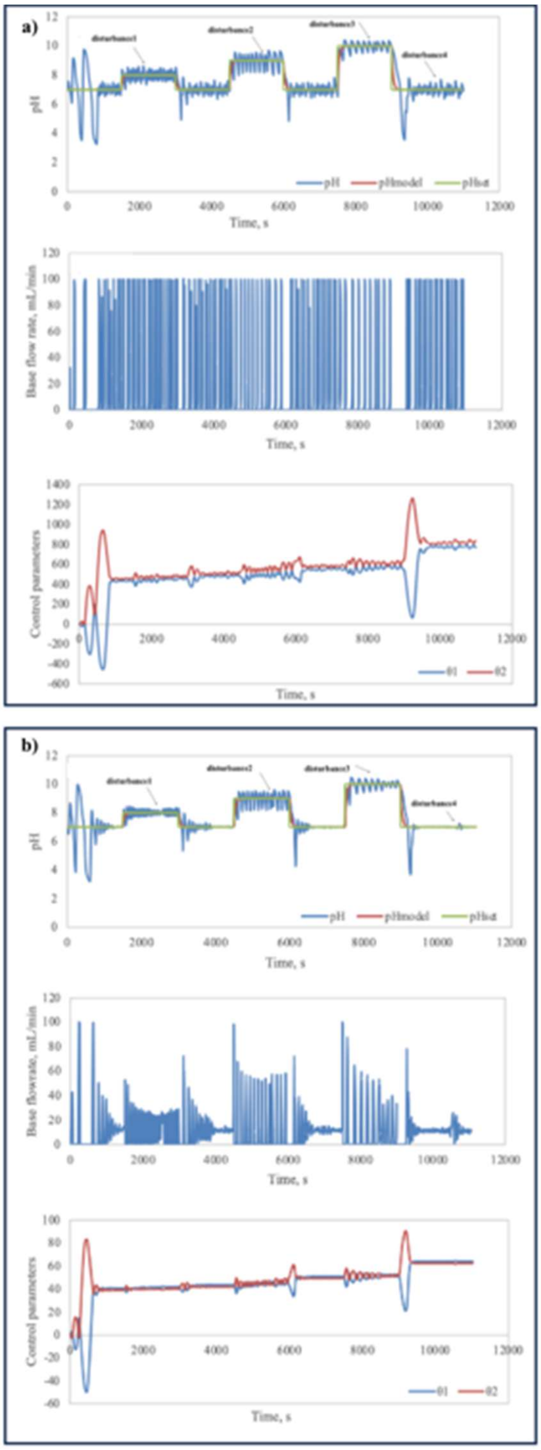

Figure 6. Experimental results for Model Reference Adaptive Control (MRAC) with variation of control parameters θ_1 and θ_2 with time (the acid flow rate which is the load effect (disturbance) was increased twice (15 mL/min to 30 mL/min) for 60 s at pH values of 7 (disturbance 1 at 2500 s), 8 (disturbance 2 at 5500 s), 9 (disturbance 3 at 8500 s), 10 (disturbance 4 at 10500 s). The upper subplot shows the controlled variable (pH response), while the lower subplot shows the manipulated variable (base flow rate) and adaptive parameters: a) γ = 0.025 and b) γ = 0.25.

Chemical processes are inherently complex, prompting continuous advancements in control algorithms to optimize product quality. Over the years, researchers have explored various control strategies, including conventional PID control [27,28], hybrid approaches integrating intelligent techniques with PID [29-31], and advanced control methodologies such as generalized minimum variance control [32,33], generalized predictive control [34,35], fuzzy control [36], neural network [37], MRAC [23,24,38,39] to enhance industrial chemical and biochemical processes.

Goud and Swarnkar [38] conducted a comparative study of PID, PID optimized with a GA, MRAC, and GA-based MRAC for a CSTR. Their findings indicated that GA-based MRAC significantly improved steady-state responses in CSTR operations [38]. Similarly, Ritonja *et al.* [39] evaluated MRAC in milk fermentation within a batch bioreactor and demonstrated that MRAC effectively ensured close tracking of the bioreactor's output [39]. Luo *et al.* [24] applied MRAC to microbial fuel cells, achieving enhanced performance and accuracy [24]. Jain *et al.* [7] designed a controller for a second-order system using MRAC with the MIT rule as the adaptive mechanism. Traditional feedback controllers often struggle with process variations caused by nonlinear actuators, environmental changes, and disturbances, leading to performance degradation in online applications. The MIT rule modifies controller parameters to minimize the cost function, which depends on the error between the plant's output and the reference model. Although this method is sensitive to variations in the amplitude of the reference signal, it yields satisfactory results [7].

In this study, the effectiveness of MRAC with the MIT rule was theoretically and experimentally compared with conventional PID control. An advanced adaptive control strategy was proposed as an alternative to traditional feedback control methods, which may be inadequate in preventing sudden pH fluctuations caused by nonlinear acid-base reaction dynamics.

One limitation of the MIT rule is its sensitivity to measurement noise and the potential risk of instability if the adaptation gain is chosen too large. In this work, these issues were mitigated by employing a low-pass filter on the pH sensor signal and by selecting γ within a conservative range (0.005-0.05). As shown in Table 1, excessively high values of γ (e.g. 0.25) indeed led to oscillations, confirming this limitation. Nevertheless, within the identified safe region, the MIT rule provided stable and robust performance in both simulation and experimental trials. Future work may explore normalized or modified MIT rules to further reduce sensitivity to noise.

CONCLUSION

This study analyzed the MRAC scheme using the MIT rule, evaluating its control performance through SIMULINK simulations and experimental results. A comparison between PID and MRAC for pH neutralization showed that increasing the adaptation gain improves system response, but exceeding a threshold ($0.05 > \gamma > 0.25$) deteriorates performance. The results emphasize the importance of selecting an appropriate adaptation gain. Within the optimal range, the MIT rule effectively ensures the system follows the reference model closely. The MRAC approach, coupled with the MIT rule, showed excellent performance and is recommended for control systems affected by setpoint variations and disturbances.

Both simulations and experiments confirmed that MRAC performs well, aligning closely with predictions regarding peak time, settling time, and Integral Square Error (ISE). The study concluded that MRAC stabilizes the system as effectively as conventional PID control. Future research recommendations include a) setting nonzero initial values for MRAC parameters θ_1 and θ_2 or using prior convergence values to improve performance; b) reducing oscillations caused by input step changes by employing the Normalized MIT rule; and c) implementing two different models to prevent parameter deviation, including a polynomial model for better pH dynamics capture.

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NAPREDNA KONTROLA NEUTRALIZACIJE PH POMOĆU ADAPTIVNE KONTROLE SA REFERENTNIM MODELOM (MRAC) I MIT PRAVILOM

Ova studija predstavlja dizajn i implementaciju adaptivnog kontrolera sa referentnim modelom (MRAC) koristeći MIT pravilo za proces neutralizacije pH u kontinuiranom reaktoru. Inherentna nelinearnost kiselinsko-baznih reakcija čini konvencionalnu PID kontrolu nedovoljnom za rukovanje brzim varijacijama pH. Da bi se ovo rešilo, predložena je strategija adaptivne kontrole, koja omogućava sistemu da dinamički podešava parametre kontrole na osnovu odstupanja u realnom vremenu od referentnog modela. Pojačanje adaptacije (γ) igralo je ključnu ulogu u stabilnosti i performansama sistema, a simulacije i eksperimentalni rezultati potvrđuju da je γ = 0,025 dalo optimalne karakteristike odziva. Veća pojačanja adaptacije ubrzala su konvergenciju, ali su uvela oscilacije, dok su niže vrednosti usporavale odziv. MATLAB/Simulink simulacije i eksperimentalna validacija u realnom vremenu pokazale su da je MRAC efikasno stabilizovao sistem, postižući brže vreme smirivanja i poboljšane performanse praćenja u poređenju sa PID kontrolom. Rezultati sugerišu da je MRAC sa MIT pravilom održiva alternativa za složene nelinearne procese, nudeći poboljšanu robusnost na poremećaje i varijacije zadatih vrednosti. Dalja poboljšanja, uključujući normalizovano MIT pravilo i polinomsko modeliranje, mogla bi dodatno usavršiti efikasnost kontrolera u industrijskim primenama.

NAUČNI RAD

Ključne reči: adaptivno upravljanje sa referentnim modelom (MRAC), MIT pravilo, pojačanje adaptacije, PID upravljanje, nelinearni sistemi, neutralizacija pH vrednosti.